



1. Solve the following equations;

(a) $x^2 y' - xy = y^3$ (7%)

(b) $xy'' + 2y' = 4x^3$ (8%)

(c) $y'' - 6y' + 9y = 8e^{3x}$ (7%)

(d) $y''' - 5y'' + 2y' + 8y = x^2$; $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$ (8%)

2. Apply the power series method to the following equation and write the solution with first five terms at least.

$$y'' + 2xy' + (x-1)y = 0; \quad y(0) = 1, \quad y'(0) = 2. \quad (10\%)$$

3. Solve the following equation by Laplace transform method.

$$y'' + 2y' + 2y = \delta(t-3); \quad y(0) = y'(0) = 0. \quad (10\%)$$

4. Response with *True* or *False* for the following statements. If it is true, then prove it. If it is false, then provide a counterexample.

(a) The set of all $n \times n$ symmetric matrices is a subspace of $\mathcal{R}^{n \times n}$. (4%)

(b) The set of all $n \times n$ nonsingular matrices is a subspace of $\mathcal{R}^{n \times n}$. (4%)

(c) Any set containing the zero vector is linearly dependent. (4%)

(d) Let V be a vector space, and let $S_1 \subseteq S_2 \subseteq V$. If S_2 is linearly dependent, then S_1 is linearly dependent. (4%)

(e) The inverse of an orthogonal matrix is an orthogonal matrix. (4%)

5. (a) For what values of a is the vector $\begin{bmatrix} a^2 \\ -3a \\ -2 \end{bmatrix}$ in $\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \right\}$? (10%)

(b) Determine whether the vector \mathbf{v} belongs to $\text{span} \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v} = \begin{bmatrix} 5 \\ 6 \\ 2 \\ 1 \end{bmatrix}; \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 5 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}. \quad (10\%)$$

(c) Let

$$A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 2 & 1 & 1 & 2 \\ 1 & -7 & 2 & -2 \end{bmatrix}.$$

Describe the set of all vectors \mathbf{b} in \mathcal{R}^3 for which the linear system $A\mathbf{x} = \mathbf{b}$ is consistent (i.e., has a solution). (10%)

y



計算題共四題

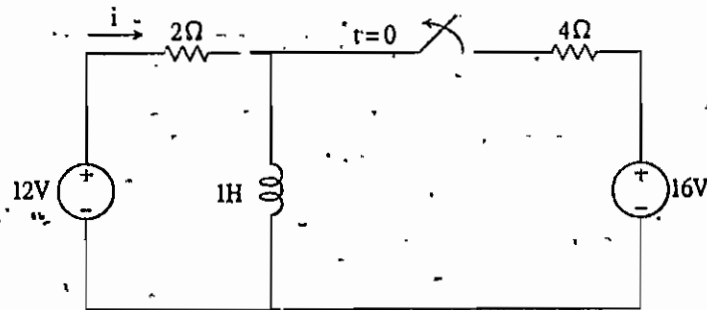
1.(25%) Before $t=0$, the circuit in Fig.1 has reach steady state, find $i(t)$ for $t > 0$.

Fig.1

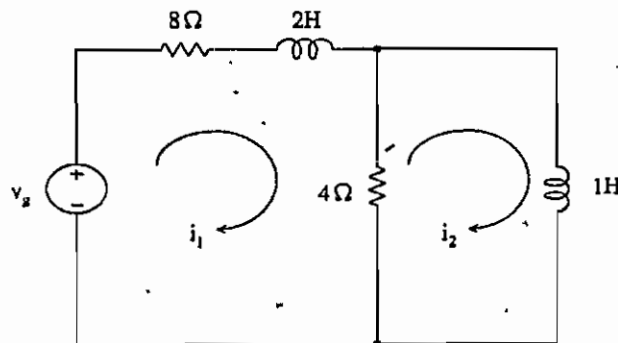
2.(25%) In Fig.2 $v_s = 16V$, find the total response of $i_2(t)$ for $t > 0$, i.e. natural response and forced response.

Fig.2



3. (25%) In the circuit of Fig. 3, determine the value of R so that the electric power absorbed by this resistance is a maximum.

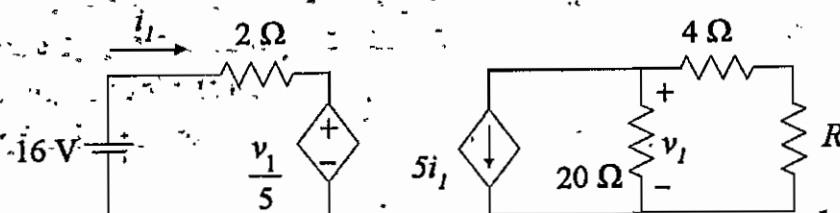


Fig. 3

4. (25%) In the circuit of Fig. 4, find the range of ω in which the current source supplies more active power than the voltage source.

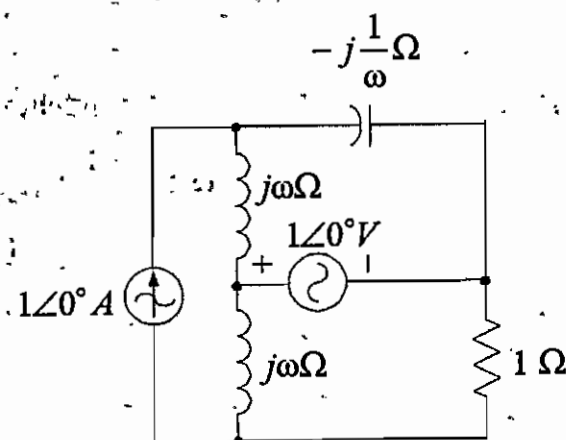


Fig. 4



1. Express (or define) the following statements. (20%)

- a. stability in the Lyapunov sense.
- b. relaxed systems in input-output system
- c. zero-input equivalence.
- d. observable systems

2. Find the solution of the dynamical system for the following system. (10%)

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [1 \ 2] \mathbf{x}(t).$$

where $u(t)$ is unit step function and $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

3. Check the controllability and observability of the following system, (20%)

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ e^{-t} \end{bmatrix} u(t)$$

$$y(t) = [1 \ 1] \mathbf{x}(t)$$

4. For unit feedback system with the forward transmittance

$$G(s) = \frac{10}{s^3 + 8s^2 + 2s}$$

determine the system type number and if the response reaches steady state, find the steady state output-input error to unit step input. (25%)

5. Ketch the root locus plot, for an adjustable constant K between 0 and $+\infty$, for the system with

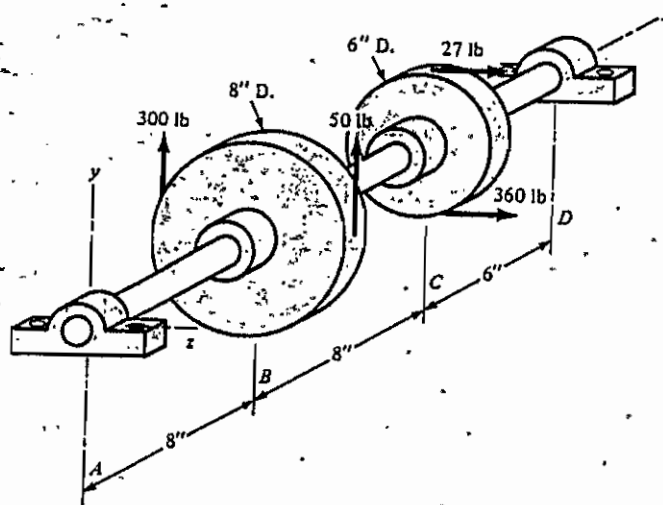
$$G(s)H(s) = -\frac{3s}{(s+2)(s^2+6s+18)} \quad (25\%)$$



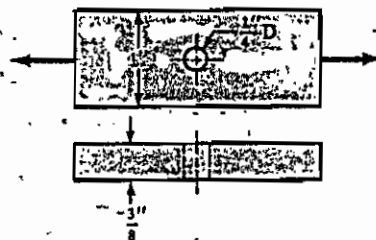
1. Write an efficient algorithm to generate a prime number greater than x and less than y , where x and y are arbitrary numbers. What is its time complexity and how much memory does it use? (25%)
2. Construct the string-matching automaton (finite state transition diagram) for the pattern $P = \text{aabab}$ and illustrate its operation on the text string $T = \text{aaababaabaababaab}$. Write an algorithm to compute the string-matching automaton given any arbitrary pattern and alphabet. What is its time complexity and how much memory does it use? (25%)
3. A set associative cache has a block size of four 16-bit words and a set size of 2. The cache can accommodate a total of 4096 words. The main memory size that is cacheable is 64K x 32bits. Design the cache structure, and show how the processor's addresses are interpreted. (25%)
4. Consider a hypothetical 32-bit microprocessor having 32-bit instructions composed of two fields: the first byte contains the op code and the remainder the immediate operand or an operand address.
 - (a) What is the maximum directly addressable memory capacity (in bytes)? (5%)
 - (b) Discuss the impact on the system speed if the microprocessor bus has (10%)
 1. a 32-bit local address bus and a 16-bit local data bus
 2. a 16-bit local address bus and a 16-bit local data bus
 - (c) How many bits are needed for the program counter and the instruction register? (10%)



1. The figure shows a shaft mounted in bearings at A and D and having pulleys at B and C. The forces shown acting on the pulley surfaces represent the belt pulls. The shaft is to be made of ASTM grade 25 cast iron with the ultimate strength of 25 ksi. If the factor of safety is 2.8 and the maximum shear stress theory is applied, what diameter should be used for the shaft? (Hint: the key point is B) [25%]



2. The cold-drawn AISI 1018 steel bar shown in the figure is subjected to a tension load fluctuating between 800 and 3000 lb. The yielding strength and the ultimate strength of this material is 54 ksi and 64 ksi, respectively. The endurance limit is 26.7 ksi and the fatigue stress concentration factor is 2.13. Estimate the factors of safety guarding against failure by yielding and by fatigue action. [25%]





3. The shaft shown in Fig. 3 is fixed at the ends. Find the reactions at the ends and the stresses in each portion of the shaft. Draw views of the elements for each portion and show the stresses acting. (25%)

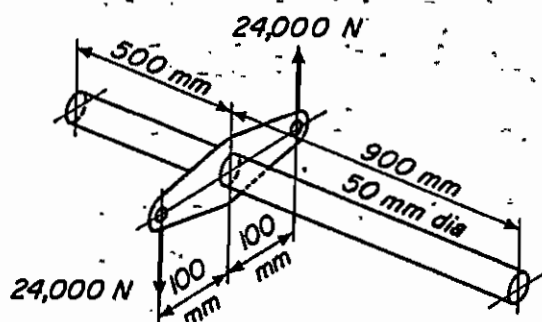


Figure 3

4. A plate clutch with a single friction surface is 250 mm OD and 100 mm ID. $\mu = 0.2$. (25%)
- If the uniform wear theory is valid, find the required axial force for $p_{\max} = 0.7$ MPa. Find the torque for the clutch. (6%)
 - Do the same for a similar clutch where the uniform pressure theory is valid for $p = 0.7$ MPa. (6%)
 - If the uniform wear theory is valid, find the torque the clutch will carry for $F_n = 22,500$ N, and the value of p_{\max} . (6%)
 - Do the same for a clutch where the uniform pressure theory is valid. (7%)



1. (1) 試說明如何提高「自動化金屬切削加工」之效率。請由刀具、加工機器、材料之輸送、監視與控制等方面，指出各個環節你所考慮到的重要特性。 [30%]
(2) 試說明當你選擇用「成形加工」製作一個工件時，你會採用某一種製程(如衝壓、鍛造以及其他等)所考慮的因素有哪些。 [20%]
2. (1) 傳統之塑膠射出成型產品肉厚約 3mm，目前發展之薄肉射出成型產品肉厚則降至 1mm 以下，如行動電話外殼等。請比較薄肉與傳統之塑膠射出成型之異同，又使用薄肉射出成型時應注意哪些可能產生之問題及如何解決？ (25%)
(2) 精密陶瓷可以所謂陶瓷粉末射出成型方式製造，請說明其製造程序，製程中可能產生之問題、產品瑕疵及解決之技術關鍵及極限為何？產品之可能應用範圍為何？ (25%)



本考卷共有三大類型，第一類為解釋名詞有四題，第二類型為說明題有四題，第三類型為計算題有二題，總分共為 100 分。

一、解釋名詞：(共 20 分)

- (1) 晶界 (Grain Boundary) (5%)
- (2) 差排 (Dislocation) (5%)
- (3) 塑性變形 (Plastic Deformation) (5%)
- (4) 活化能 (Activation Energy) (5%)

二、說明題：(共 55 分)

- (1) 請說明延性轉脆溫度(Ductility Transition Temperature), 並請說明其在材料設計時所扮演的角色。 (20%)
- (2) 請說明材料受外力作用的變形行為, 並請說明其變形機構。 (20%)
- (3) 請解釋為何多晶形材料 (polycrystalline materials) 之特性通常為等方向性 (isotropic)? (5%)
- (4) 在下列各種不同原子之電子組態 (electron configurations) 中, 請決定其分別為惰性氣體 (inert gas)、鹵素 (halogen)、鹼金屬 (alkali metal)、鹼土金屬 (alkaline earth metal) 或過渡金屬 (transition metal) 原子之電子組態, 並請說明理由。 (10%)
 - (a) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^7 4s^2$
 - (b) $1s^2 2s^2 2p^6 3s^2 3p^6$
 - (c) $1s^2 2s^2 2p^3$
 - (d) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

三、計算題：(共 25 分)

- (1) 試證明面心立方晶體 (the face-centered cubic crystal) 之原子密集度 (atomic packing factor, APF) 為 0.74。 (15%)
- (2) 某一合金含有 33 g 銅與 47 g 鋅, 請問各元素之原子百分比 (atom percent, at%)? 其中銅之原子量為 63.54 g/mole, 鋅之原子量為 65.37 g/mole. (10%)



3. (a) It is common knowledge that the temperature of air rises as it is compressed. An inventor thought about using this high-temperature air to heat buildings. He used a compressor driven by an electric motor. The inventor claims that the compressed hot-air system is 25% more efficient than a conventional heating system. (13%)

1. (25%)

A well-insulated vessel is divided into two equal compartments by a partition. One compartment initially contains a gas which obeys the van der Waals equation of state and the other compartment is initially evacuated. The partition is removed and the gas allowed to equilibrate. Show that the final temperature of the gas is given by $T_f = T_i - a/(2C_v V_i)$, where "a" is a constant in the van der Waals equation and V_i is the initial volume that the gas occupies. (Two respective locations are 24°C and 4°C. Mean as the heat source and sink? Why? (13%))

Note: van der Waals equation $(P + a/V^2)(V - b) = RT$.

2. (25%)

Ethane undergoes a throttling expansion, during which its temperature falls from 200 °F to 162 °F. Ethane is not ideal under the process conditions, but its behavior is adequately represented by $P = RT/(V - b)$, where $b = 1.028 \text{ ft}^3/\text{lbmole}$. Over this temperature range the mean heat capacity at constant pressure may be regarded as constant at 13.3 Btu per lbmole-°F. What is the pressure drop in psia during this expansion? (5%)

(1 Btu = 5.40 ft³-psia)

(c) When apply superheating to the vapor power cycle, both the net work and heat input increase. Why it still can increase the cycle's thermal efficiency? (5%)

(d) For a fixed turbine inlet temperature under superheating, what is the effect of increasing the boiler pressure? What kind of side effects will it cause? Any suggestions to correct the problems? (10%)



Fig. 1



3. (a) It is common knowledge that the temperature of air rises as it is compressed. An inventor thought about using this high-temperature air to heat buildings. He used a compressor driven by an electric motor. The inventor claims that the compressed hot-air system is 25% more efficient than a resistance heating system that provides an equivalent amount of heating. Is this claim valid? Explain. (12%)

(b) In tropical climates, the water near the surface of the ocean remains warm as a result of solar energy absorption. However, the water in the deeper parts of the ocean still remains at a relatively low temperature. If the water temperature at the two respective locations are 24°C and 4°C , is it worthy to construct a power plant and use the ocean as the heat source and sink? Why? (13%)

4. Fig.1 is an ideal T-s diagram of a vapor power cycle. Please answer the following question:

(a) Use the T-s diagram to indicate the amount of heat supplied, heat rejected and work output. (5%)

(b) On the T-s diagram, indicates the deviations of an actual power cycle from the ideal one, when irreversibilities and pressure losses in pipes are considered. (5%)

(c) When apply superheating to the vapor power cycle, both the net work and heat input increase. Why it still can increase the cycle's thermal efficiency? (5%)

(d) For a fixed turbine inlet temperature under superheating, what is the effect of increasing the boiler pressure? What kind of side effects will it cause? Any suggestions to correct the problems? (10%)

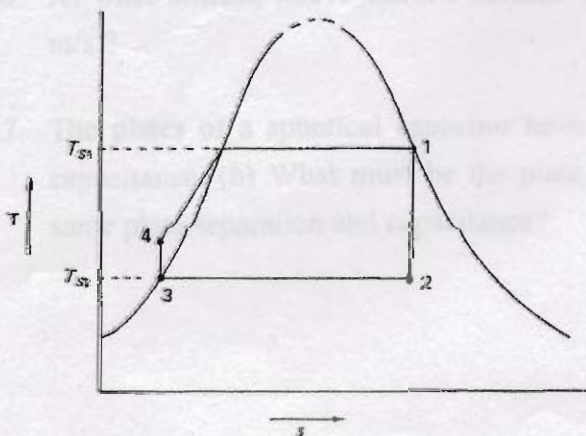


Fig.1



下列各題，每題 10 分

- State Le Chatelier's principle.
 - How will an increase in pressure affect the equilibrium, solid \rightleftharpoons vapor?
- The standard cell potential $E^0 = 0.6365 \text{ V}$ for the galvanic cell

$$\text{Zn (s)} \mid \text{Zn}^{2+} (0.500\text{M}) \parallel \text{Pb}^{2+} (? \text{M}) \mid \text{Pb (s)}$$

The potential of the cell was measured to be 0.4438 V .

What was the molar concentration of Pb^{2+} in its half-cell?

$$\log 0.5 = -0.301$$
- Define *half-life of a reaction*.
 - How is the half-life of a first-order reaction affected by the concentrations of the reactants? Please use equations for description.
- What is meant by the term *effective nuclear charge*?
 - What is the lanthanide (La) contraction?
- A 0.2000-g sample of a fishy smelling liquid known to contain only carbon, hydrogen, and nitrogen was burned and produced 0.482 g CO_2 and some water. A second sample weighing 0.2500 g was treated in such a way that all the nitrogen in the substance was converted to N_2 . This gas was collected and found to occupy 42.3 mL at 27°C and 755 torr .

 - What are the percentages of carbon, hydrogen, and nitrogen in the compound?
 - What is the empirical formula of the compound?

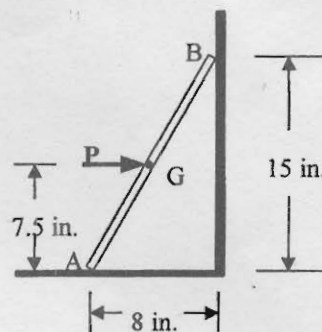
The gas constant : $0.0821 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$
- At what altitude above Earth's surface would the gravitational acceleration be 4.9 m/s^2 ?
- The plates of a spherical capacitor have radii 38.0 and 40.0mm . (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?



8. A controller on an electronic arcade game consists of a variable resistor connected across the plates of a $0.220 \mu\text{F}$ capacitor. The capacitor is charged to 5.00V , then discharged through the resistor. The time for the potential difference across the plates to decrease to 0.800V is measured by a clock inside the game. If the range of discharge times that can be handled effectively is from $10.0 \mu\text{s}$ to 6.00ms , what should be the resistance range of the resistor?
9. The shortest wavelength emitted by a bat is about 3.3mm . What is the corresponding frequency?
10. A 200 turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.30A . Calculate the magnetic field \mathbf{B} inside the solenoid.



- 一、施加一水平力 P 使 10 磅重之均勻桿 AB 維持於圖示之位置，已知 A 及 B 處之摩擦係數為 0.2，求使桿保持平衡之最小力 P 。〔20 分〕



- 二、一個懸臂支撐之平面混凝土擋水牆，高 4 ft，厚 1 ft，若混凝土的比重為 145 lb/ft³，水的比重為 62.4 lb/ft³，(a)當水高達牆頂時，求牆底處之最大及最小應力，(b)若欲使混凝土中無張應力，則水之最大允許深度 d 為何？〔20 分〕
- 三、兩根圓形斷面之樑，其截面積相同，若第一根樑為實心（直徑 d_1 ），第二根樑為空心（外徑 d_2 ），試求兩根樑的斷面模數比 S_2/S_1 。〔10 分〕
- 四、仔細說明何謂應力、應變與楊氏係數。〔25 分〕

- 五、拉力實驗求得某材料之應力與應變對應數據如下：

應力	1	2	3	4	5	6	7	8
應變	2.2	3.9	6.1	7.8	10.1	11.7	12.1	12.5

- 如何求得該材料之楊氏係數？〔25 分〕