國立雲林科技大學
94學年度博士班招生入學考試試題

所別：工程科技研究所
科目：工程數學（甲）

1. Given a system of equations as below (25%)

$\dot{x}_1 = x_1 + x_2$ *with* $x_1(0) = 2$

$\dot{x}_2 = 3x_1 - x_2$ $\quad x_2(0) = -2$

(1) Please find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

(2) Solve the above system of equations **by using the eigen values and eigen vectors obtained in (1).**

2. Please draw the time function curve in Cartesian coordinates (i.e. f(t) vs. t) and find the **Laplace Transformation** of following functions. (25%)

Note that u(t) is the unit step function.

(1) $f(t) = e^{-4t}u(t)$

(2) $f(t) = e^{-4(t-2)}u(t-2)$

(3) $f(t) = e^{-4t}u(t-2)$

(4) $f(t) = e^{-4(t-2)}u(t)$

(5) $f(t) = e^{-4(t+2)}u(t)$

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3. 設 $\vec{f} = z^2\vec{k}$ ，求面積分 $\iint_S \vec{f}.\hat{n}dS = ?$ (25%)

(1) 以直接面積分求之

(2) 以 divergence 定理求之 ($\iint_S \vec{f}.\hat{n}dS = \iiint_V \nabla \cdot \vec{f}dV$)

其中 S:S_1+S_2, 圓錐體底面在 xy 平面，半徑爲 1 的圓。圓錐頂在(0,0,1)，即圓錐高爲 1

S_1: 圓錐體側面，令 $\vec{R}$ 爲 S_1 上任一點位置向量

$$\vec{R} = (1-z)\cos\theta\vec{i} + (1-z)\sin\theta\vec{j} + z\vec{k}$$

S_2:圓錐體底面，在 xy 平面半徑爲 1 的圓

4. Solve the heat problem for the temperature function (25%)

T(x, t): $\dfrac{\partial T}{\partial t} = k\dfrac{\partial^2 T}{\partial t^2}$ in the $0 \le x \le 1$ $t > 0$, k is constant

with the boundary condition: $\dfrac{\partial T}{\partial x}(0,t) = 0$ $T(1,t) = 0$

and the initial condition: $T(x,0) = \begin{cases} 1 & 0 \le x \le \dfrac{1}{2} \\ 2(1-x) & \dfrac{1}{2} \le x \le 1 \end{cases}$

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1. (10%) Let $f(x)=x^2$ be integrable with respect to both $g_1(x)=x$ and $g_2(x)=\lfloor x \rfloor$ on the closed interval [0, 5]. What is $\int_0^5 f(x)d(g_1(x)+g_2(x))=?$ (Note that $g_2(x)$ is a floor function, for example, $g_2(\pi)=\lfloor \pi \rfloor = 3$.)

2. (10%) Prove that there does not exist a rational number r such that $r^2=2$.

3. (10%) Let $T: R^5 \to R^4$ be defined by $T(\mathbf{x}) = A\mathbf{x}$, where $\mathbf{x}$ is in R^5 and

$$A=\begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$$

Find a basis for the kernel of T as a subspace of R^5.

4. (10%) Suppose computer operators at terminals only do one type of job, reading and thinking and typing followed by waiting for a response for the computer. Let the operator's average time for reading and thinking and typing be $\boldsymbol{T_{think}}$ and the average response time of the computer for one command request from an operator be $\boldsymbol{T_{system}}$. Suppose there are N operators and one computer. Please give the upper bound and the lower bound of the total mean throughput rate for this operators-computer system model.

5. (10%) Find the eigenvalues of $A=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$ and find a basis for each of the corresponding eigenspaces.

6. (10%) Find all the eigenvalues λ of the matrix $A=\begin{bmatrix} 4 & 3 & 0 & 0 & 0 \\ -2 & -1 & 0 & 0 & 0 \\ 1 & 7 & 10 & 1 & -7 \\ 2 & -1 & 0 & 5 & 0 \\ -3 & 1 & 6 & -4 & -3 \end{bmatrix}$

7. (10%) Calculate the line integral $\oint_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = (3-2y)\mathbf{i} + (3x-4y)\mathbf{j} + (z+3y)\mathbf{k}$, and C: $x^2+y^2=1,\ z=2$. ($\mathbf{i}, \mathbf{j}, \mathbf{k}$) represents the three unit vectors in Cartesian coordinate system.

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8. (15%) Find the solution: $y'' + (x+2)y' + (x+1)y = 0$; $y(0) = a$, $y'(0) = 0$.

9. (15%) Solve the partial differential equation: $\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} + x + y + 1 = 0$;

$u(0,0) = u_x(x, 0) = u_y(0, y) = 0$.

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1. 求解微分方程式 $y'+\frac{4}{x}y=2$；$y(1)=-4$。 (15%)

2. 求解微分方程式 $x^2y''-3xy'+4y=0$； $y(1)=4,\quad y'(1)=5$。 (15%)

3. 試求 $F(s)=\dfrac{2s-3}{s^2+s-2}$ 的逆變換 $f(t)$。 (10%)

4. 試求 $F(s)=\dfrac{3s-2}{s^2+4s+20}$ 的逆變換 $f(t)$。 (10%)

5. 一週期函數 $f(x)$ 其定義爲 $f(x)=\dfrac{x^2}{4},\quad -\pi<x<\pi$ 且 $f(x+2\pi)=f(x)$
   (i) 試求其傅立葉級數(Fourier series)。 (12%)
   (ii) 求 $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots=?$ (8%)

6. 傅立葉變換(Fourier transform)定義爲 $F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-i\omega t}\,dt$，求函數

   $F(\omega)=\dfrac{4e^{(3\omega-6)i}}{5-(2-\omega)i}$ 之逆傅立葉變換(inverse Fourier transform) $f(t)$。 (10%)

7. 令 $v_1=\begin{bmatrix}1\\3\\-2\end{bmatrix},\ v_2=\begin{bmatrix}-2\\-6\\4\end{bmatrix},\ v_3=\begin{bmatrix}1\\2\\t\end{bmatrix}$. 求所有的 t 值使得向量 v_3 能由 $\{v_1\quad v_2\}$

   所生成(span)。 (10%)

8. 在平面上有三個 $(x,\ y)$ 點分別爲 $(1,\ 0)$，$(2,\ 3)$ 及 $(3,\ 9)$。試求以 $y=c_0+c_1x$ 最佳近似此三點。 (10%)

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請從後面三個附件中，選讀其中之一，然後針對所選讀的附件回答下列各問題。

(附件1結構工程與材料領域：第2至10頁、附件2營建管理領域：第11至17頁、附件3建築領域：第18至25頁)。請清楚標明所選讀之附件編號。

選讀附件1者，依下列四點評論：

(1) 研究目的與主要貢獻。(25分)

(2) 研究方法與特色。(25分)

(3) 參考文獻Fig. 2，說明為何「矩形橫箍柱」圍束效果比「圓形螺箍柱」差。(25分)

(4) 試推導現行柱圍束箍筋之設計公式。(25分)

選讀附件2或3者，依下列四點評論：

(1) 本篇文獻之背景與研究目的。(25分)

(2) 本篇文獻之研究方法。(25分)

(3) 本篇文獻之具體貢獻。(25分)

(4) 本篇文獻之缺點與限制。(25分)

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ACI STRUCTURAL JOURNAL TECHNICAL PAPER

Title no. 99-S1

附件一

Displacement-Based Design of Reinforced Concrete Columns for Confinement

by Murat Saatcioglu and Salim R. Razvi

A displacement-based design procedure was developed for confinement of earthquake-resistant concrete columns. The procedure is based on experimentally observed and analytically computed relationships among the parameters of confinement. The amount, grade, spacing, and arrangement of transverse reinforcement; concrete strength and cover thickness; and the level of axial compression and drift ratio were considered as parameters of confinement. Static inelastic (pushover) analyses were conducted to generate a large volume of data, with due considerations given to concrete confinement, reinforcement strain hardening and buckling, anchorage slip, axial compression, and secondary deformations due to P-Δ effect. Both normal-strength and high-strength concrete columns with circular and square cross sections were included. Improved design expressions were developed for column confinement utilizing both the current design criterion, which is based on column axial deformability, and the recommended design criterion, which is based on lateral deformability as expressed by column drift ratio.

Keywords: column; ductility; confined concrete; displacement-based design; high-strength concrete; transverse reinforcement.

INTRODUCTION

Reinforced concrete columns built in seismically active regions are expected to undergo a large number of inelastic deformation cycles while maintaining overall strength and stability of the structure. This can be ensured by proper confinement of the core concrete. The confinement requirements of the ACI 318-99 Building Code[1] provide satisfactory designs in most applications.[2-4] The same requirements, however, may also result in unsatisfactory designs, leading to either unsafe or overconservative columns, which often lead to the congestion of reinforcement and related construction problems.[2-4] The code requirements were derived for normal-strength concrete columns and are not applicable to columns cast from high-strength concrete.

The state of knowledge on concrete confinement has improved substantially since the pioneering work of Richart, Brandtzaeg, and Brown[5,6] in 1928, which formed the basis for the ACI 318 design requirements.[1] During the last three decades, a large volume of experimental data has been generated, and a number of improved analytical models have been developed that describe the stress-strain behavior of confined concrete. A better understanding of design parameters has also been acquired, including those that are currently overlooked by the design practice. Test data have become available on high-strength concrete columns, making it possible to develop design provisions for such columns that are not currently included in the ACI 318 building code.[1] It is the objective of this paper to present improved design expressions for normal- and high-strength concrete column confinement on the basis of the current state of knowledge.

CURRENT DESIGN APPROACH

The design criterion adopted in ACI 318-99[1] for column confinement is based on the premise that confined columns should maintain their concentric capacities after the spalling of cover concrete. This is achieved by providing sufficient confinement to the core concrete to attain strength and ductility enhancements. The required volumetric ratio of transverse reinforcement was derived based on the strength gain in core concrete, which was assumed to be $(f'_{cc} - f'_{co}) = 4.1 f_\ell$, where f_ℓ represents uniform passive confinement pressure.[6] Equating the concentric capacity of cover concrete to the strength gain in core, the required volumetric ratio of transverse reinforcement that satisfies the ACI 318 performance criterion may be obtained as follows

Strength in cover concrete = strength gain in core concrete

$$0.85 f_c'(A_g - A_c) = 4.1 f_\ell (A_c - A_s) \tag{1}$$

The lateral pressure f_ℓ for a spirally reinforced circular column at yield is

$$f_\ell = \frac{2A_{sp} f_{yh}}{s h_c} \tag{2}$$

Substituting f_ℓ into Eq. (1) and dividing both sides by $(2.05 f_{yh} A_c)$

$$0.415 \frac{f_c'}{f_{yh}}\left(\frac{A_g}{A_c} - 1\right) = \frac{4A_{sp}}{s h_c} - \frac{4A_{sp}A_s}{s h_c A_c} \tag{3}$$

$$\frac{4A_{sp}}{s h_c} = \frac{4\pi h_c A_{sp}}{\pi h_c^2 s} = \rho_s = 0.415 \frac{f_c'}{f_{yh}}\left(\frac{A_g}{A_c} - 1\right) + \frac{4A_{sp}A_s}{s h_c A_c} \tag{4}$$

Equation (4) was adopted by ACI 318[1] after dropping the last term and increasing 0.415 to 0.45. The code expression is shown in Eq. (5) for spirally reinforced circular columns where hoop tension results in near-uniform lateral pressure, which is consistent with the degree of strength enhancement considered.

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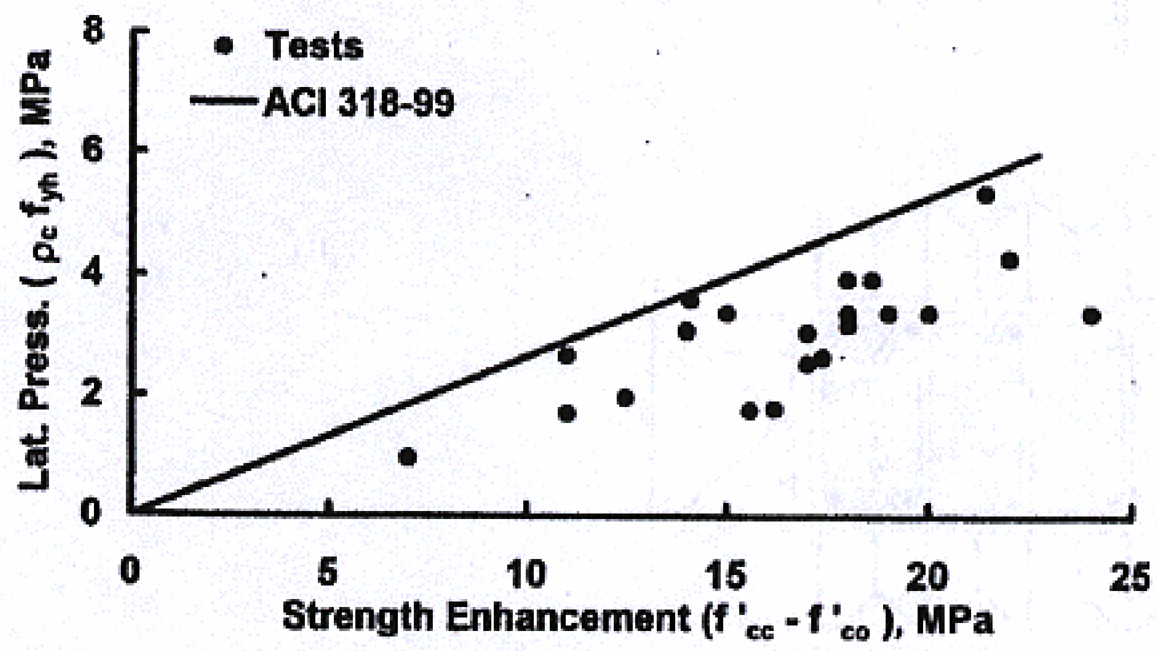
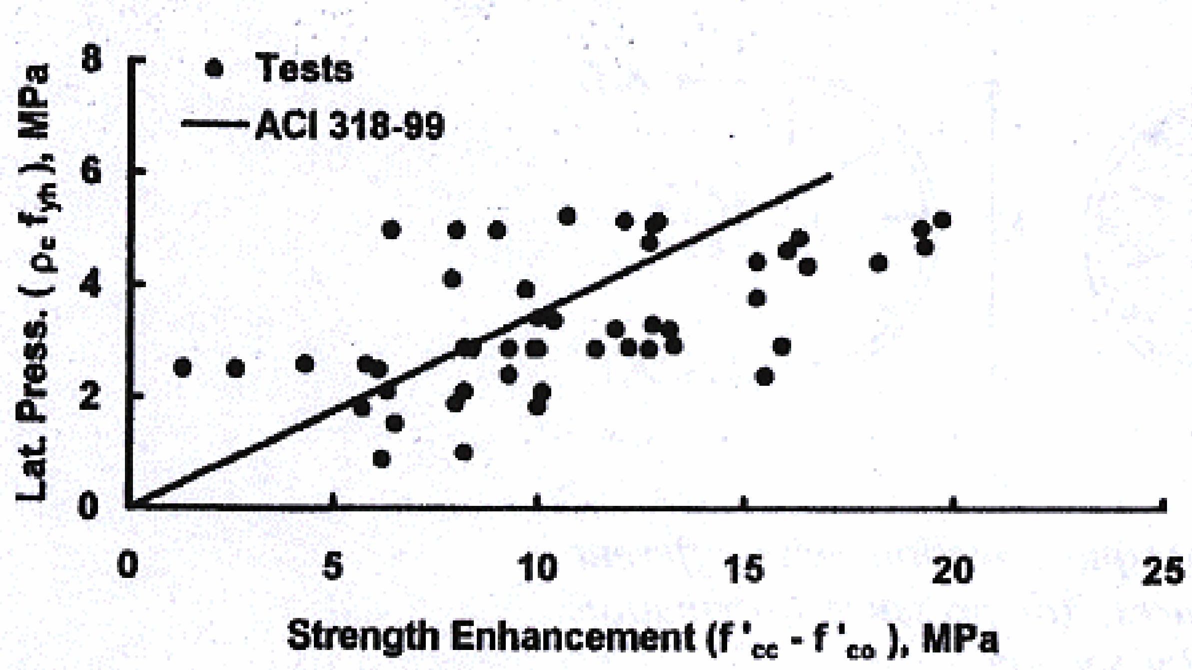
Salim R. Razvi *is an assistant professor at UAE University, United Arab Emirates. He received his MASc and PhD from the University of Ottawa. His research interests include column confinement and performance of high-strength concrete columns.*



(a) Circular columns



(b) Square columns

Fig. 1—Comparisons of experimental data with ACI 318 requirements.[1]

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}} \quad (5)$$

With the aforementioned simplification, the strength enhancement in core becomes $(f'_{cc} - f'_{co}) \approx 3.8f_\ell$. The lateral pressure f_ℓ for a spirally reinforced circular column can be written in terms of the area ratio of transverse reinforcement ρ_c in each cross-sectional direction

$$f_\ell = \frac{2A_{sp}f_{yh}}{b_c s} = \rho_c f_{yh} = \frac{1}{2}\rho_s f_{yh} \quad (6)$$

The strength enhancement based on ACI 318 is compared with experimental values obtained by Sheikh and Toklucu[7] and Mander, Priestley, and Park[8] in Fig. 1(a). The comparison indicates that Eq. (5) does not provide a good correlation with experimental data, producing over-conservative quantities of transverse reinforcement for spirally reinforced circular columns. This is attributed to the constant multiplier 3.8 used in relating strength gain to confinement pressure, although test results reported in the literature indicate a variable multiplier that is a function of lateral pressure f_ℓ.[9,10]

In large columns, the ratio of cross-sectional area to confined core area (A_g/A_c) may approach unity. In this case, Eq. (5) results in very small values of volumetric ratio. Therefore, a lower-bound expression is provided by setting a limit on the A_g/A_c ratio. This translates into Eq. (7)

$$\rho_s = 0.12\frac{f'_c}{f_{yh}} \quad (7)$$

The confinement steel requirements for square and rectangular columns are based on an arbitrary extension of the aforementioned requirements, while recognizing that rectilinear reinforcement is not as effective as circular reinforcement. The code expression for the required area of rectilinear reinforcement is obtained from Eq. (5), based on the premise that rectilinear reinforcement is 3/4 as effective as circular spirals. This implies that 1/3 more steel is needed in square and rectangular columns to attain deformabilities usually expected from spirally reinforced circular columns. The increased steel requirement, when expressed in terms of the area of lateral reinforcement, translates into Eq. (8) with the corresponding strength enhancement of $(f'_{cc} - f'_{co}) \approx 2.8f_\ell$. The lower limit established is similar to that for circular spirals, and is shown in Eq. (9)

$$A_{sh} = 0.3sh_c\frac{f'_c}{f_{yh}}\left(\frac{A_g}{A_c} - 1\right) \quad (8)$$

$$A_{sh} = 0.09sh_c\frac{f'_c}{f_{yh}} \quad (9)$$

The previously mentioned requirements of the ACI 318 building code[1] are compared with the results of concentrically tested columns, obtained by Sheikh and Uzumeri,[11] Scott, Park, and Priestley,[12] Razvi and Saatcioglu,[13] and Abdulkadir,[14] in terms of lateral pressure $\rho_c f_{yh}$ and the resulting strength enhancement. The comparison, shown in Fig. 1(b), indicates poor correlation. This may be explained mostly by the differences in behavior resulting from different arrangements of reinforcement, which is a parameter that is not considered in Eq. (8). Researchers in the past showed that columns with the same amount and spacing of confinement reinforcement showed significantly different strength and deformability when confined by different arrangements of transverse reinforcement.[11,12,15,16] While a square column with four corner bars and tied with perimeter hoops shows the worst behavior, columns confined with well-distributed longitudinal reinforcement, laterally supported by crossties, overlapping hoops, or both, show significantly improved performance.[11,12,15,16] It is clear from the comparisons shown in Fig. 1 that the code expressions do not provide adequate representation of experimental observations for the performance criterion for which they were developed.

PROPOSED DESIGN APPROACH

A design procedure is proposed in this paper based on the confinement model developed by the authors.[16] Initially, the performance criterion used in the ACI code is adopted, while recognizing important interactions among the design parameters that are currently overlooked in ACI 318-99.[1] More specifically, the tradeoff between the volumetric ratio and reinforcement arrangement is considered. Tie spacing and

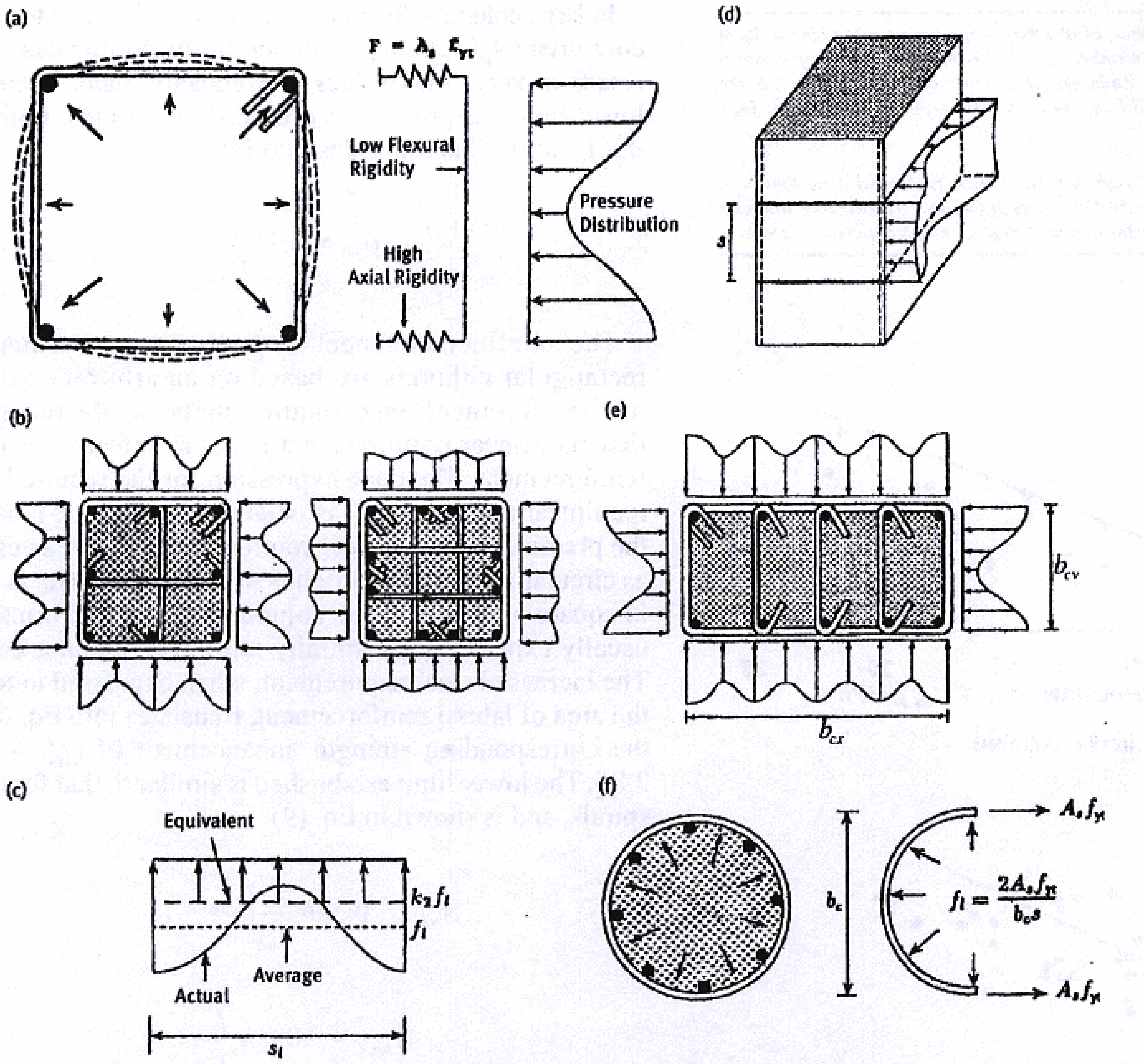


Fig. 2—Development of confinement pressure:[16] (a) and (b) square sections with different arrangements; (c) actual, average, and equivalent pressures; (d) pressure distribution along column height; (e) rectangular section; and (f) circular section.

spacing of laterally supported longitudinal reinforcements are explicitly addressed. Confinement of high-strength concrete, with strength of up to approximately 130 MPa, is included. It is shown that the volumetric ratio of confinement reinforcement can be reduced for columns with efficient tie arrangements. The treatment of square and rectangular columns is particularly improved since the confinement steel requirements are based on a realistic analytical model, reflecting experimental observations rather than an arbitrary extension of the concepts derived for circular columns. The following expressions define strength enhancement in confined concrete based on the analytical model adopted

$$f'_{cc} = f'_{co} + k_1 k_2 f_\ell \quad (10)$$

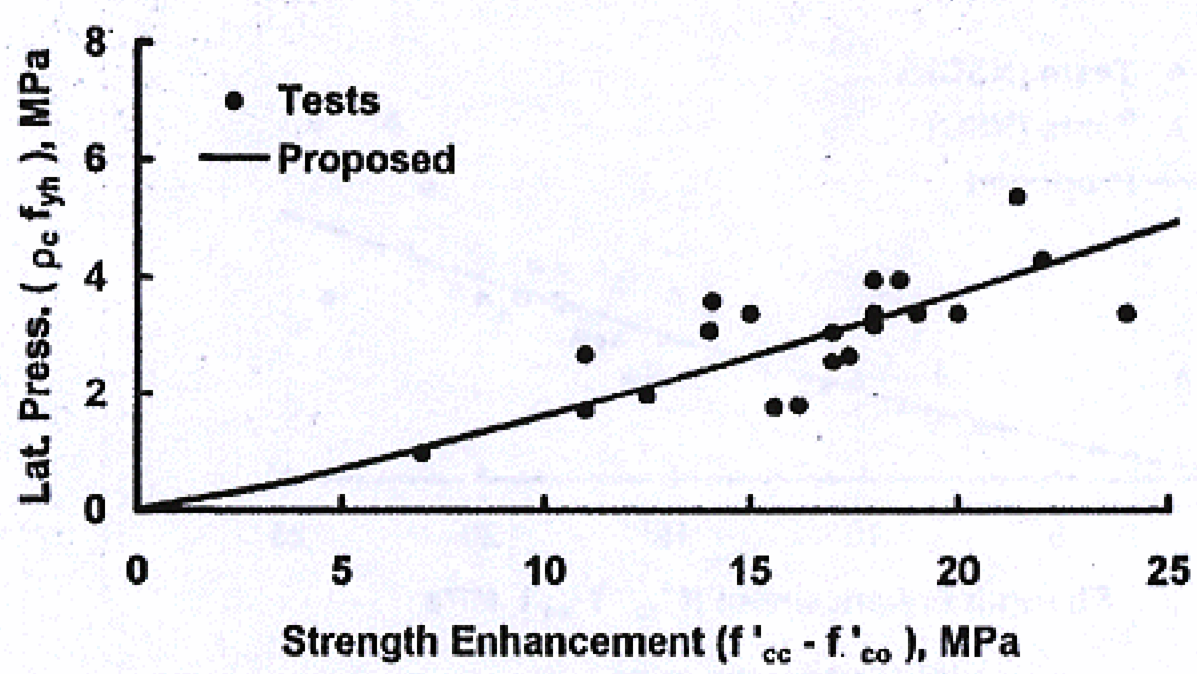
$$f_\ell = \frac{\sum A_s f_{yh}}{s b_c} \quad (11)$$

$$k_1 = 6.7(k_2 f_\ell)^{-0.17} \quad (12)$$

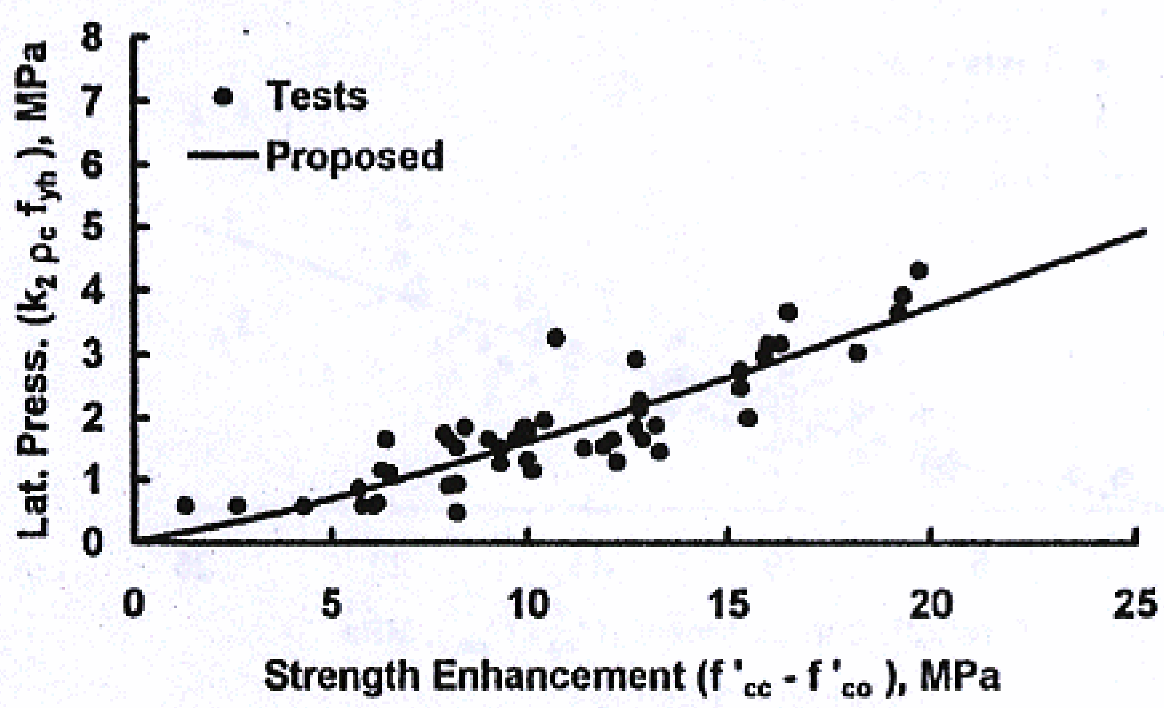
$$k_2 = 0.26\sqrt{\frac{b_c}{s}\frac{b_c}{s_\ell}\frac{1}{f_\ell}} \quad (13)$$

where f_ℓ in Eq. (12) and (13) is in MPa. The in-place strength of concrete f'_{co} may be taken as equal to $0.85 f'_c$, as per ACI 318-99,[1] and also as supported by test data. Coefficient k_1 reflects the relationship between uniform lateral confinement pressure and strength enhancement. This coefficient was found to vary with lateral pressure in previous experiments,[6,9,10] differing substantially from the constant value of 4.1 used in deriving the ACI 318 expressions. Coefficient k_2 relates the average lateral pressure f_ℓ to equivalent uniform pressure, and reflects the efficiency of confinement reinforcement. The efficiency improves with the uniformity of confinement pressure and reaches its full value when the lateral pressure is uniform, as is approximately the case in circular columns with closely spaced spirals for which $k_2 = 1.0$.

The passive confinement pressures generated by different arrangements of rectilinear reinforcement are illustrated in Fig. 2. As can be seen, the restraining action against lateral expansion becomes high at locations of cross-reinforcement where overlapping hoops, crossties, or both are tied to the longitudinal reinforcement. Hence, both the tie spacing s as well as the spacing of cross-reinforcement in the cross-sectional plane s_ℓ play important roles in the efficiency of reinforcement arrangement. These variables are incorporated into Eq. (13). A simplified version of the same equation, also applicable to high-strength concrete, has been suggested by the authors and is shown in Eq. (14)[17]



(a) Circular columns



(b) Square columns

Fig. 3—Comparisons of experimental data with proposed equation.

$$k_2 = 0.15\sqrt{\frac{b_c b_c}{s\ s_\ell}} \tag{14}$$

Equation (10) through (14) can be used to derive new design expressions while maintaining the performance criterion adopted by ACI 318-99.[1] This criterion requires the concentric capacity of confined column core to be at least equal to the unconfined strength of the entire column section. Ignoring the area of concrete replaced by longitudinal reinforcement, the resulting expression can be written as follows

$$f'_{cc}A_c = f'_{co}A_g \tag{15}$$

$$(0.85f'_c + k_1k_2f_\ell)A_c = 0.85f'_cA_g \tag{16}$$

$$\frac{k_1k_2f_\ell}{0.85f'_c} = \frac{A_g}{A_c} - 1 \tag{17}$$

$$\frac{8(k_2\rho_cf_{yh})^{0.83}}{f'_c} = \frac{A_g}{A_c} - 1 \tag{18}$$

$$\rho_c = 0.0825\frac{(f'_c)^{1.2}}{f_{yh}}\frac{1}{k_2}\left(\frac{A_g}{A_c} - 1\right)^{1.2} \tag{19}$$

Equation (19) incorporates the parameters of confinement that play important roles on axial deformabilty, including the arrangement of reinforcement. It provides the required area ratio of transverse confinement reinforcement in each cross-sectional direction for circular, square, and rectangular sections. For circular spirals as defined in ACI 318-99,[1] k_2 = 1.0. For all other cases, k_2 can be computed using Eq. (14).

The confinement steel requirement specified by Eq. (19) is verified against experimental data in terms of lateral pressure $k_2\rho_cf_{yh}$ and the resulting strength enhancement in Fig. 3. The figure indicates that significant improvements are achieved by the proposed expression over the expressions given in ACI 318-99[1] and shown in Fig. 1.

For the majority of columns in practice, Eq. (19) may be simplified as follows

$$\rho_c = 0.2\frac{f'_c}{f_{yh}}\frac{1}{k_2}\left(\frac{A_g}{A_c} - 1\right) \tag{20}$$

High-strength concrete columns

Recent research on high-strength concrete columns indicates that the strength gain due to confinement is independent of concrete strength, although the percentage of strength gain becomes lower for higher-strength concretes.[18-20] Therefore, high-strength concrete columns require proportionately more confinement to attain deformabilities usually expected from earthquake-resistant columns. Tests on high-strength concrete columns also reveal that higher-grade reinforcement is effective in confining columns.[18-20] It was shown by the authors that the effectiveness of high-grade confinement steel under concentric compression depended on the amount and efficiency of transverse steel, while it also depended on the level of axial compression for columns subjected to lateral load reversals.[19-22] Transverse reinforcement with yield strengths of up to 1000 MPa was found to be effective under monotonically increasing concentric compression when confined to conform to Eq. (19).[19,20] The same steel was effective under lateral deformation reversals when the accompanying level of axial load was approximately 40% P_o. The transverse steel was approximately 80% effective when the level of axial load was reduced to 20% P_o, developing approximately 800 MPa stress at peak column resistance.[22] Reinforcement with approximately 600 MPa yield strength was consistently effective in confining high-strength concrete columns. Therefore, until further experimental data become available, it may be prudent to limit the yield strength of transverse reinforcement in Eq. (19) and (20) to 600 MPa, which provides an increase of approximately 50% in the current limit of 400 MPa used in ACI 318.[1]

The applicability of Eq. (19) to high-strength concrete columns is verified against experimental data. Figure 4 shows the comparison of strength enhancement values obtained by tests and by Eq. (19) for both normal-strength and high-strength concrete columns. The test data on high-strength concrete were obtained by different researchers for concrete strengths ranging between 30 and 124 MPa.[20,21,23-28] The lateral pressure for these columns was based on recorded transverse steel stress at peak column resistance f_s whenever available, rather than yield strength, since the high-grade reinforcement used in some of the columns did not necessarily yield, which confirms the validity of the upper limit suggested in the previous paragraph. The comparison indicates good correlations with test data providing experimental evidence on the applicability of Eq. (19) to high-strength concrete columns.

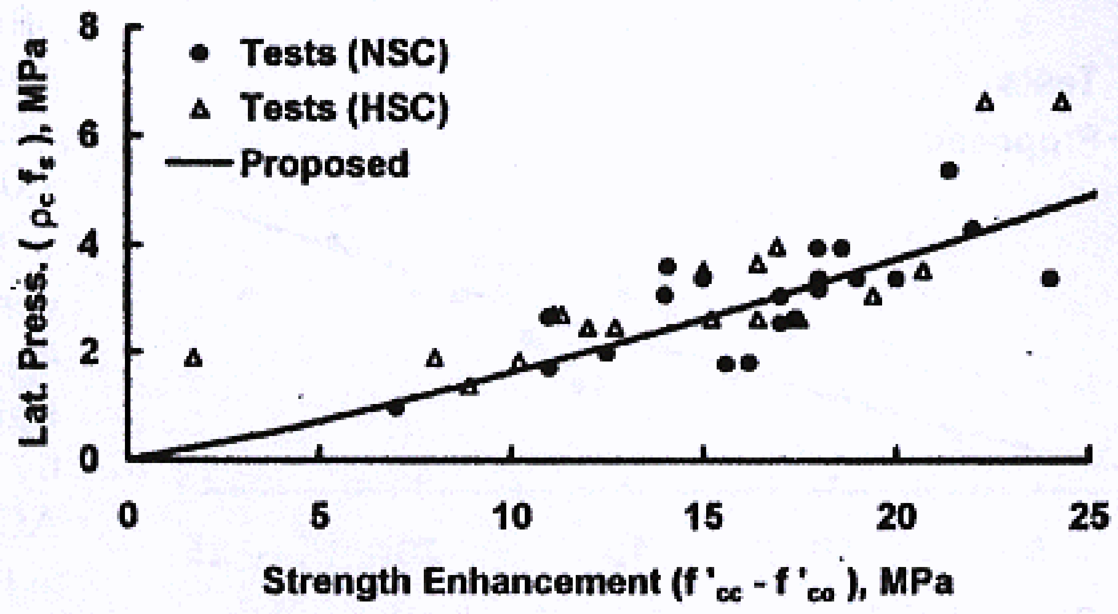
Displacement-based design

The design requirements discussed in the preceding section are based on the axial deformability of columns under concentric compression, and conform to the ACI 318[1] design criterion. This design criterion, however, is not representative of actual column behavior during seismic response. Columns of building and bridge structures experience lateral drift when subjected to seismic excitations. It has been shown by previous research that there is a direct correlation between lateral drift and concrete confinement.[29] Consequently, columns that experience significant lateral drift should be confined more stringently that those that are braced laterally by rigid structural walls. Lateral drift is not explicitly addressed in ACI 318-99[1] for column confinement. Instead, the confinement requirements were developed on the basis of axial deformability, with the implied understanding that columns deformable under concentric compression are also deformable under combined axial and lateral loading. This criterion does not permit the level of axial compression and/or the drift demand to be introduced as design parameters.

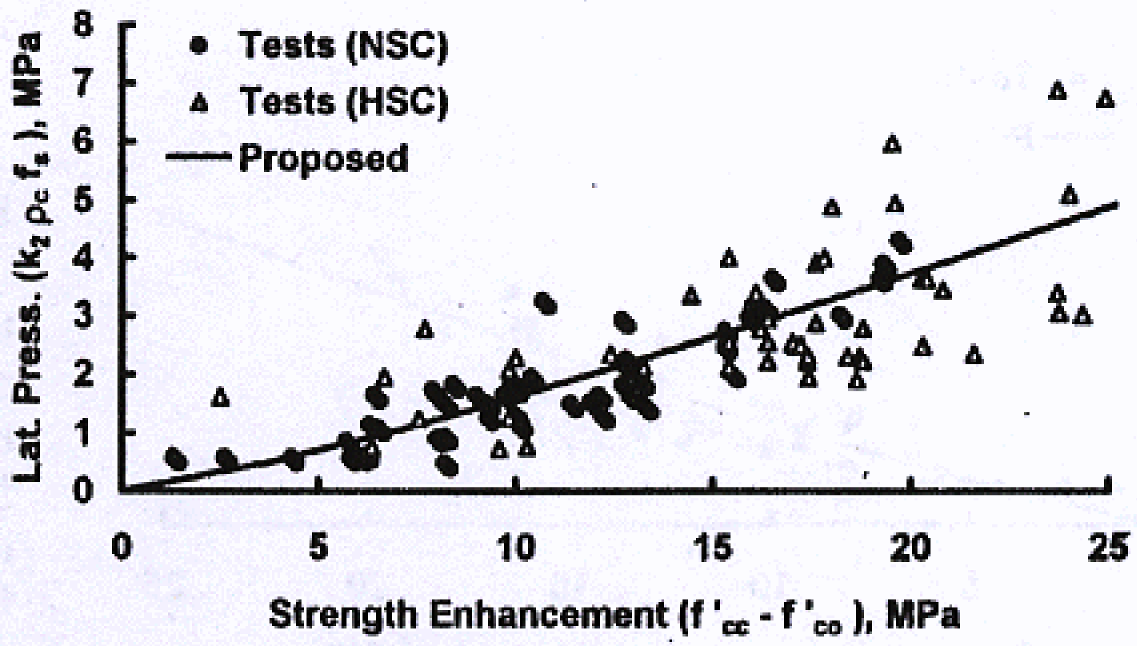
A displacement-based design approach is presented in this section, with lateral drift as the performance criterion. The design approach is based on computed drift capacities of columns with different levels of confinement and axial compression. The computation of drift was done using a computer program for static inelastic loading (pushover analysis)[30] that incorporates analytical models for concrete confinement,[16,17] steel strain hardening,[30] bar buckling,[30] formation and progression of plastic hinging,[29] and anchorage slip[31,32] (extension of reinforcement in the adjoining member). The analysis procedure also includes an option for second-order deformations caused by P-Δ effects. The analytical models, as well as the analysis procedure employed, had been verified extensively against experimental data.[16,17,29,31-33] Figure 3, 4, 5 and 6 illustrate sample comparisons of computed and measured response for the entire range of inelastic force-deformation relationships for both normal-strength and high-strength concrete columns. These sample comparisons, as well as those reported elsewhere,[27,29,37] provide experimental verification of the analysis procedure employed in deriving the design expressions.

The drift capacity was computed either at 20% strength decay in moment resistance or at the same level of decay in lateral force resistance. In the latter case, the decay included the portion that was caused by the P-Δ effect. The use of 20% strength decay as the failure criterion is consistent with that employed by previous researchers, since it is reasonable to accept some strength decay in columns of multistory multibay structures during seismic response before they can be considered to have failed.[3,38]

Extensive parametric investigation was conducted to establish the significance of design parameters on lateral drift.[29,37] The results were used to identify primary design parameters for column confinement while establishing relationships between axial load, confinement reinforcement, and drift capacity. It was concluded that the amount, grade, spacing, and arrangement of confinement reinforcement, as well as the level of axial compression, concrete strength, cover-core area ratio, and shear span-depth ratio played important roles on drift capacity, while the percentage of longitudinal reinforcement played a role of secondary importance. It was further concluded that similar drift capacities could be obtained from columns with similar geometry and reinforcement arrangement but different amounts of



(a) Circular columns



(b) Square columns

Fig. 4—Comparison of normal-strength concrete (NSC) and high-strength concrete (HSC) column tests with proposed equation.

confinement reinforcement and material strengths, as long as the $\rho_c f_{yh}/f_c'$ ratio remained constant, with certain limits placed on these design parameters. This indicates that the $\rho_c f_{yh}/f_c'$ ratio could be used as a design parameter for a wide range of material strengths, including high-strength concrete and high-grade reinforcement. Further verification of this point was done experimentally for concrete strengths up to 124 MPa and steel strengths up to 1000 MPa.[19-22] It was also established that the relationship between the required level of confinement and cover-core area ratio was approximately linear within the practical range of 0.2 to 0.8. Consequently, it was confirmed that columns having a constant $\rho_c f_{yh}/\{f_c'[(A_g/A_c) - 1]\}$ ratio would develop approximately similar drift capacities when other confinement parameters remained constant, irrespective of variations in individual parameters that make up this ratio.[29,37] This ratio, defined as r, is used in establishing the confinement steel requirements, as also used by ACI 318-99.[1]

$$r = \frac{\rho_c f_{yh}}{f_c'\left[\frac{A_g}{A_c} - 1\right]} \tag{21}$$

Figure 7 and 8 illustrate the variation of column drift capacity with coefficient r, defined in Eq. (21) for different levels of axial compression and efficiency of transverse reinforcement k_2. Drift ratios plotted in Fig. 7(a) and 8(a) were determined at 20% strength decay in lateral force capacity; these account for the decay in force resistance caused by the P-Δ effect. Hence, they are lower than those shown in Fig. 7(b) and 8(b), which were determined at 20% strength decay in moment capacity. These figures clearly indicate that

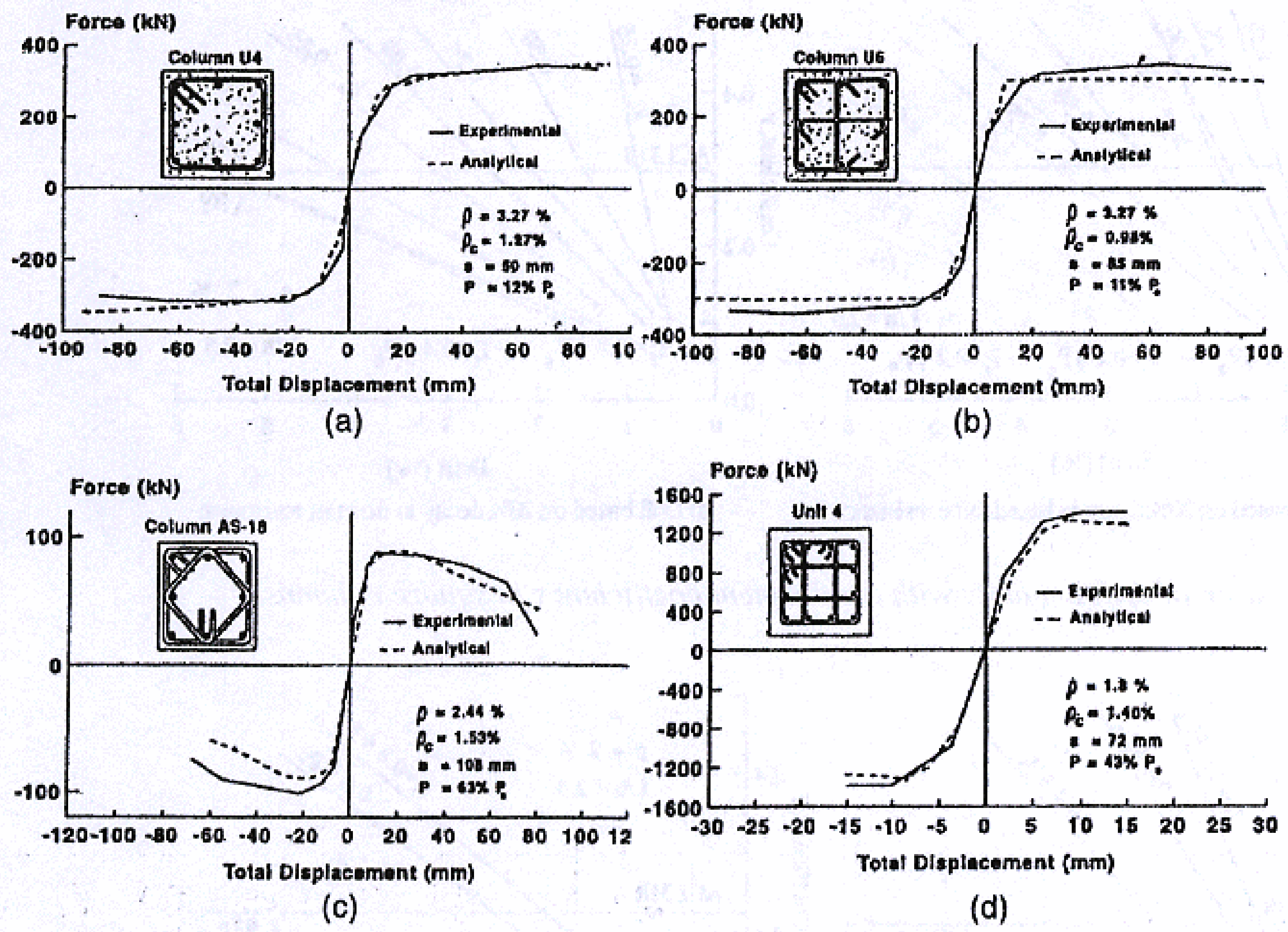


Fig. 5—Normal-strength concrete columns tested by: (a) and (b) Saatcioglu and Ozcebe;[34] (c) Sheikh and Khoury;[35] and (d) Park, Priestley, and Gill.[36]

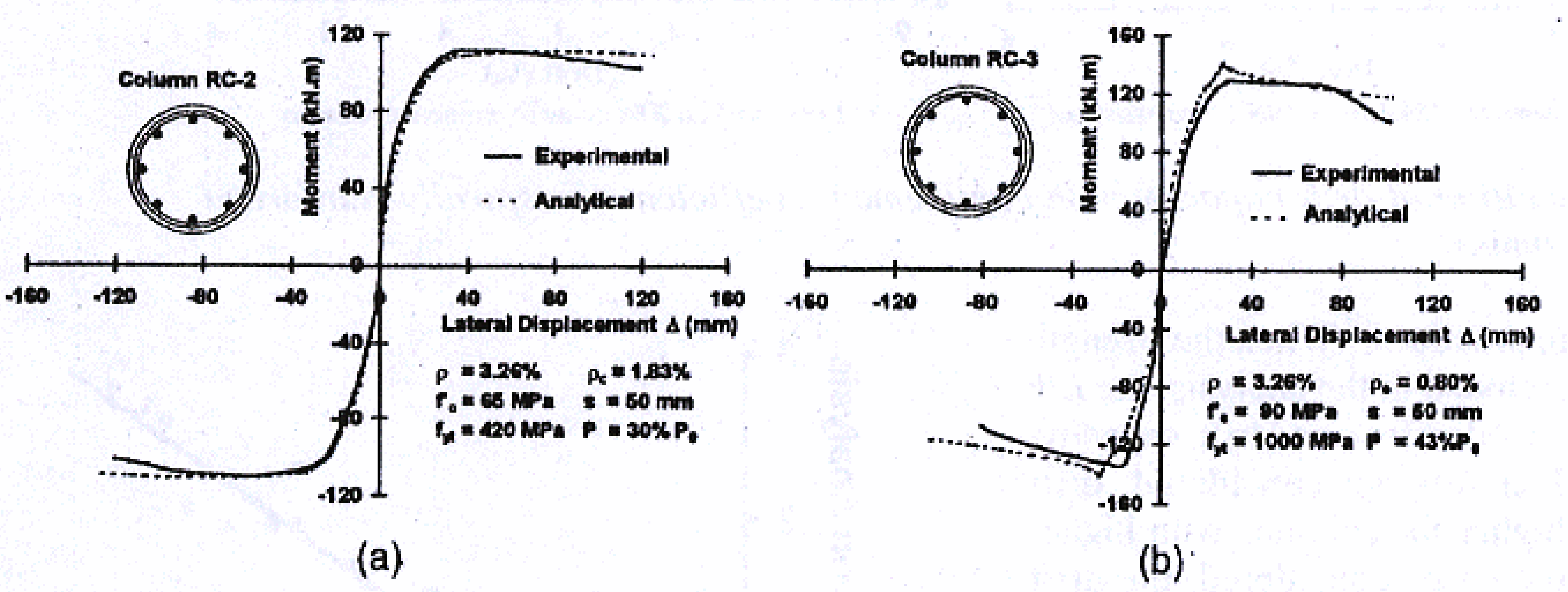


Fig. 6—High-strength concrete columns tested by Saatcioglu and Baingo.[22]

the lateral drift capacity (deformability) improves with increasing values of coefficient r and the efficiency of reinforcement arrangement k_2. They further indicate that the column drift capacity decreases with increasing axial compression. Therefore, higher percentage and/or higher grade and/or improved efficiency of transverse reinforcement are required for columns under higher compression. This implies that the confinement requirements may be relaxed for columns under lower levels of axial compression. Figure 7 and 8 also suggest that the confinement steel requirements should not only be a function of axial load level, but also the arrangement of reinforcement k_2. The relationships given in these figures suggest that the following approximation can be made between r and lateral drift ratio δ

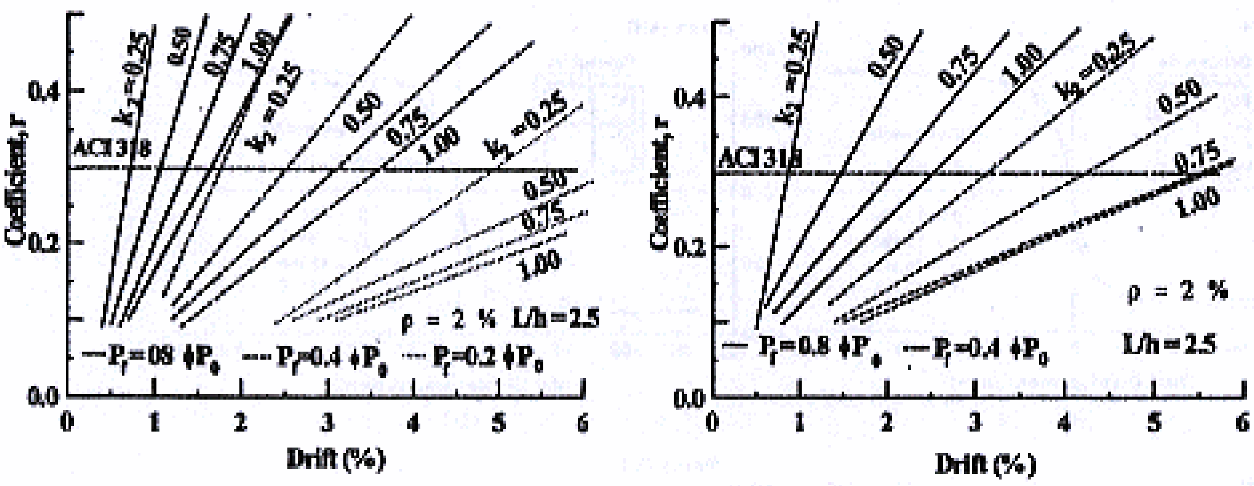
$$r = 14\frac{1}{\sqrt{k_2}}\frac{P}{P_o}\delta \qquad (22)$$

Substituting the value of r from Eq. (21) and solving for reinforcement ratio ρ_c

$$\rho_c = 14\frac{f_c'}{f_{yh}}\left[\frac{A_g}{A_c}-1\right]\frac{1}{\sqrt{k_2}}\frac{P}{P_o}\delta \qquad (23)$$

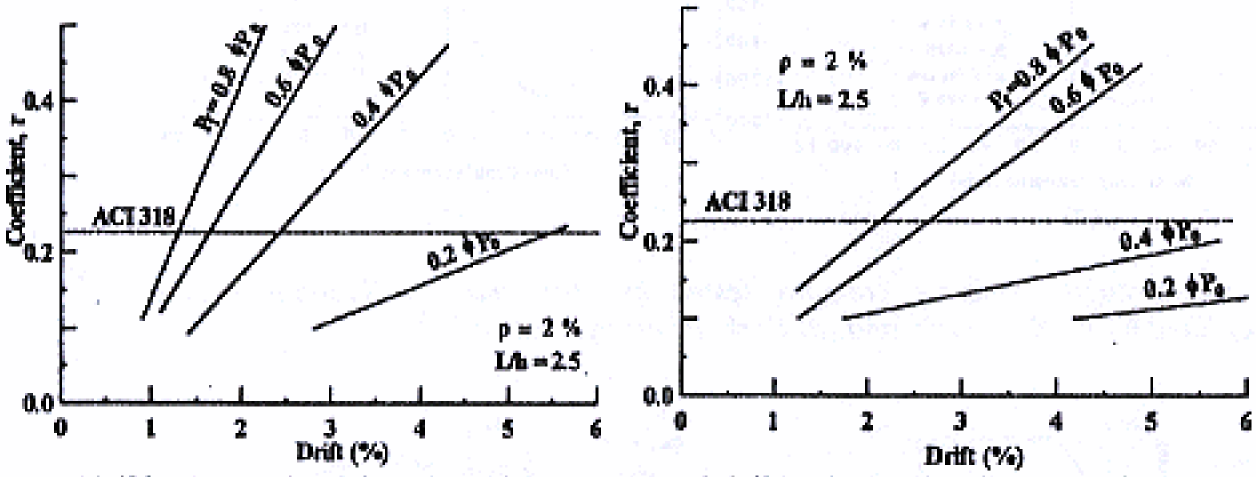
Equation (23) relates the confinement parameters to drift capacity δ in the direction of confinement reinforcement when $P \geq 0.2P_o$. Figure 9 illustrates the correlation between drift capacities obtained by Eq. (23) and inelastic pushover analyses. Since the computed drift has been verified extensively against experimental data within the entire range of inelastic drift, as indicated previously and illustrated in Fig. 5 and 6, the computed drift values may be viewed as close representations of experimental values. Figure 9 indicates that Eq. (23) provides a good estimate of column drift capacity. Hence, it can be used to establish the confinement steel requirements of columns with different levels of drift demand.

Figure 7 and 8 were generated for columns with a shear span-depth ratio L/h of 2.5. This level is near the lower end of L/h ratios used in practice. A complete set of analyses was also conducted for columns with $L/h = 5.0$, representing the



a) Drift based on 20% decay in lateral force resistance b) Drift based on 20% decay in moment resistance

Fig. 7—Variation of drift capacity with confinement coefficient r in square columns.



a) Drift based on 20% decay in lateral force resistance b) Drift based on 20% decay in moment resistance

Fig. 8—Variation of drift capacity with confinement coefficient r in spirally reinforced circular columns.

higher end of the range used in practice.[37] When the strength decay due to P-Δ effect was included in the analysis, the L/h ratio did not show a pronounced effect on drift capacity. However, when the P-Δ effect was not considered, drift capacities were consistently higher for columns with higher shear span-depth ratios. In the cases considered, the drift capacity increased by approximately 75%, from $L/h = 2.5$ to $L/h = 5.0$. For design purposes, it is conservative to consider the aspect ratio that produces lower estimates of drift capacity. Hence, the results for $L/h = 2.5$ were used in developing the design expression given in Eq. (23).

The percentage of longitudinal reinforcement was also observed to have an influence on drift capacity.[37] This was expected because the increase in longitudinal steel content would increase the contribution of steel as a ductile material to overall column response and produce an increase in column deformability. Column analyses were conducted for 1, 2, and 4% longitudinal reinforcement. The results showed minor variations in drift capacity, with columns having higher percentage of longitudinal reinforcement exhibiting slightly higher drift capacities. The improvement obtained by doubling the amount of reinforcement was approximately 10%. Hence, the longitudinal reinforcement ratio ρ was not included as a parameter for confinement design. Instead, the results for an average reinforcement ratio of 2% were adopted.

The allowable story drift ratio (drift demand) specified by current building codes is limited to 2.0 to 2.5% for most concrete frame structures.[39-41] While Eq. (23) may be used for different drift ratio limits up to 4%, an expression

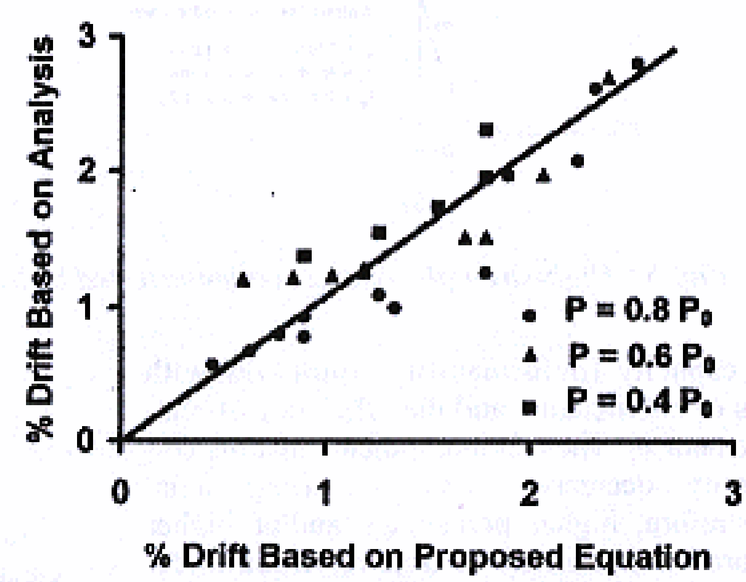


Fig. 9—Correlation of pushover analysis results with Eq. (23).

may be developed for a permissible drift ratio limit of 2.5%. When this drift level is substituted into Eq. (23), and the axial force ratio P/P_o is replaced with $P_u/\phi P_o$, a design expression can be derived as follows

$$\rho_c = 0.35\frac{f'_c}{f_{yh}}\left[\frac{A_g}{A_c}-1\right]\frac{1}{\sqrt{k_2}}\frac{P_u}{\phi P_o} \qquad (24)$$

The axial force P_u in the aforementioned expression represents the maximum axial compressive force that can possibly be applied on the column during a strong earth-

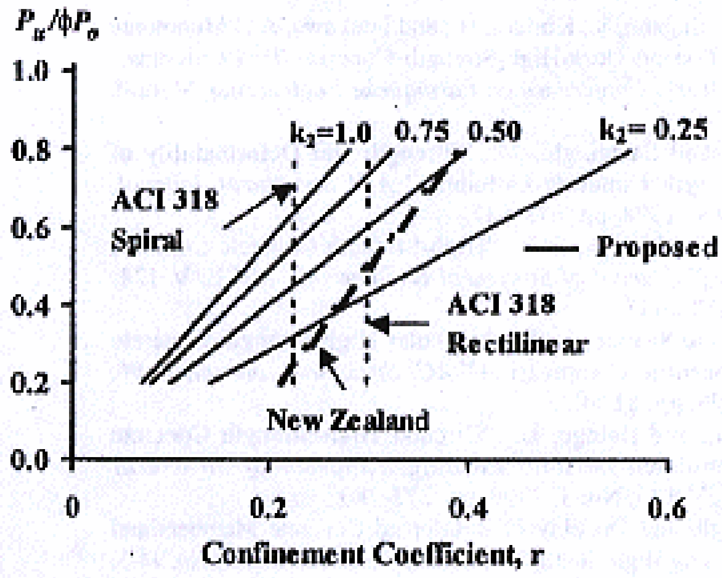


Fig. 10—Comparison of proposed confinement requirements with current American and New Zealand practices.

quake. This quantity corresponds to factored design axial compressive force in ACI 318 design practice.[1] When the capacity design approach is used, as in the case of the New Zealand practice,[42] P_u is computed at the formation of probable moment resistances at the ends of the framing beams when plastic hinges have formed at these locations. The capacity reduction factor ϕ may be taken as 0.90, as opposed to the 0.70 and 0.75 currently recommended for tied and spiral columns in ACI 318 because of the improved ductility of properly confined columns. Equation (24) provides the area ratio of transverse reinforcement required in each cross-sectional direction. For circular spirals, ρ_c remains the area ratio of spiral reinforcement, which is the same in any one direction. The reinforcement ratio requirement given in the same equation approaches zero as the axial compression approaches zero. Therefore, a lower limit is placed on the design axial compressive force, illustrated as follows

$$\frac{P_u}{\phi P_o} \geq 0.2 \tag{25}$$

Furthermore, as discussed previously, it was concluded in the parametric study[37] that the use of the cover-core area ratio as a design parameter had limitations. Therefore, the following limit, also used in ACI 318,[1] may be placed on this ratio

$$\frac{A_g}{A_c} - 1 \geq 0.3 \tag{26}$$

Comparisons with current practice

Equation (24), which is based on the proposed displacement-based design procedure, is compared with the requirements of ACI 318-99[1] and the New Zealand Code NZS 3101.[42] The confinement steel requirements of ACI 318 and NZ 3101 are reproduced as follows in terms of the area ratio of transverse reinforcement ρ_c

ACI 318-99 (spiral)

$$\rho_c = 0.225\frac{f_c'}{f_{yh}}\left[\frac{A_g}{A_c} - 1\right] \tag{27}$$

ACI-318-99 (rectilinear)

$$\rho_c = 0.3\frac{f_c'}{f_{yh}}\left[\frac{A_g}{A_c} - 1\right] \tag{28}$$

NZS 3101 (1982)

$$\rho_c = 0.3\frac{f_c'}{f_{yh}}\left[\frac{A_g}{A_c} - 1\right]\left[0.5 + \frac{1.25P_e}{\phi f_c' A_g}\right] \tag{29}$$

The comparison is made by using the confinement coefficient r that is obtained by dividing both sides of the previous equations by $f_c'/f_{yh}[A_g/A_c - 1]$. The axial force ratios of $\{P_u/\phi P_o\}$ and $\{1.25P_e/\phi f_c' A_g\}$ in Eq. (24) and (29), respectively, are approximately equal for a longitudinal column reinforcement ratio of 3% and can be assumed to be the same for the purpose of comparison. Figure 10 provides the comparison of proposed displacement-based design approach with current North American and New Zealand practices. The comparison indicates that the ACI 318[1] approach, which is not a function of the level of axial compression, produces overconservative designs for spirally reinforced columns and some columns with rectilinear reinforcement, especially when the level of axial compression is low. For columns with poor reinforcement arrangement (low k_2 value), the ACI 318 requirements can be unsafe when the axial load level is above approximately 40% of column concentric capacity P_o. The New Zealand approach recognizes the effect of axial load, but ignores the effect of reinforcement arrangement. It produces overconservative designs relative to the proposed approach for columns with superior arrangements of reinforcement (high k_2). Often, overconservative designs translate into the congestion of reinforcement cage and concrete placement problems.

SUMMARY AND CONCLUSIONS

Design expressions were developed for confinement steel requirements of earthquake-resistant concrete columns. Two different performance criteria were adopted for this purpose: 1) the ACI 318 criterion based on axial deformability; and 2) a displacement-based design criterion based on lateral drift. Design expressions were developed for both performance criteria. The expression for the latter criterion is based on static inelastic (pushover) analysis of columns, which was verified experimentally. The proposed expressions incorporate the effects of reinforcement arrangement and higher strength of steel and concrete, and also incorporate the effect of axial force for a displacement-based design. The expressions provide significant improvements over the existing practice, as evidenced by experimental verifications.

NOTATION

- A_c = area of core concrete within perimeter transverse reinforcement (center-to-center, except in Eq. (5), (9), (27), and (28), where it is measured out-to-out)
- A_g = gross area of column concrete section
- A_r = area of longitudinal steel reinforcement, except in Fig. 2 and Eq. (11), where A_r is defined as either A_{sh} or A_{sp}
- A_{sh} = area of transverse reinforcement within spacing s and perpendicular to dimension h_c
- A_{sp} = area of spiral reinforcement
- h_c = core dimension, center-to-center of perimeter tie
- d_s = diameter of spiral reinforcement
- f_c' = concrete cylinder strength
- f_{cc}' = strength of confined core concrete
- f_{co}' = in-place strength of unconfined concrete in column ($f_{co}' \sim 0.85 f_c'$)
- f_l = passive lateral confinement pressure provided by reinforcement
- f_s = stress in transverse steel at peak column resistance
- f_{yh}, f_{yt} = yield strength of transverse reinforcement

h = column sectional dimension
h_c = core dimension perpendicular to transverse reinforcement under consideration (center-to-center of perimeter reinforcement)
k_1 = lateral pressure coefficient, defined in Eq. (12)
k_2 = confinement efficiency parameter, defined in Eq. (14)
L = column shear span
P = axial compressive force on column
P_e, P_u = maximum axial compressive force on column during earthquake
P_o = nominal concentric compressive capacity of column
r = confinement coefficient, defined in Eq. (21)
s = center-to-center spacing of transverse reinforcement along column height
s_ℓ = center-to-center spacing of longitudinal reinforcement, laterally supported by corner of hoop or hook of crosstie
δ = lateral drift ratio, defined as horizontal displacement divided by height
ϕ = capacity reduction factor
ρ = longitudinal reinforcement ratio
ρ_c = area ratio of transverse confinement reinforcement, $\rho_c = A_{sh}/h_c s$
ρ_s = volumetric ratio of transverse reinforcement

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附件二

PROBABILISTIC OPTIMAL-COST SCHEDULING

By Leroy J. Isidore[1] and W. Edward Back[2]

ABSTRACT: During the planning and execution of construction projects, it often becomes necessary to shorten the duration of the project. A widely used technique for reducing the duration of a project is commonly referred to as least-cost scheduling. This procedure is based on deterministically arriving at the shortest project duration for the minimum cost possible. There is, however, one major problem with the typical application of this technique. It does not address the variability inherent in the duration and cost of the project activities. Thus, the resulting compressed schedule value cannot be applied with any stated level of statistical confidence. This paper presents a new procedure that addresses some of the major shortcomings of least-cost scheduling. It does so by accounting for the variability inherent in the duration and cost of the scheduled activities by simultaneously applying range estimating and probabilistic scheduling to the historical data. The resulting data set is then analyzed to provide a compressed schedule duration and cost estimate that have a higher overall confidence of being achieved.

INTRODUCTION

When planning the execution of construction projects, it is often necessary to address situations where it is crucial to accelerate the completion date of a project in order to finish the project earlier than was initially scheduled. Such situations can arise for several reasons. Some of these reasons include

- The opportunity to maximize the profit margin associated with the execution of a project by optimizing resource allocation and utilization and reducing project overhead cost
- The acquisition of additional projects that will compete for the limited resources of a contractor should their execution begin prior to the substantial completion of existing projects
- A request by the owner of a project to have the project delivered earlier than was initially agreed upon
- The desire to avoid inclement weather, or similar impacts, that could significantly delay the project

One common industry method of achieving this goal is through a technique commonly referred to as least-cost scheduling. This technique involves incrementally reducing (or crashing) the respective activity durations of a particular project in order to shorten the project length and arrive at the shortest schedule duration with minimum cost. Such a schedule is generally referred to as the least-cost schedule. Stephens (1990) defines a least-cost schedule as one with an optimal duration so that lengthening or shortening it would increase the total project cost.

SCHEDULE COMPRESSION AND SCHEDULE REDUCTION

Two key procedures for reducing the duration of the project are schedule compression and schedule reduction. The Construction Industry Institute (CII) undertook several detailed research studies that dealt specifically with identifying and evaluating schedule compression and reduction techniques. In one of the earlier research studies concluded in 1988 (CII 1988), a research team defined more than 90 different techniques for schedule compression. Some of these techniques resulted in shorter schedule time while others were intended to prevent needless loss of time. Of the techniques identified, not all were expected to reduce cost and time simultaneously, and in some instances, it would clearly be a time-cost trade-off. It is emphasized that the applicability of each compression technique is highly dependent on the particular situation.

In 1995, another CII research project (CII 1995) reexamined the schedule compression techniques previously compiled. This research team then made the following distinction between schedule compression and schedule reduction.

- Schedule compression is the use of techniques that shorten the project duration, which results in an increase in project cost.
- Schedule reduction is the use of techniques that shorten the project duration, which does not result in an increase in project cost.

The accepted means of arriving at a least-cost schedule is through the application of schedule compression techniques. This is accomplished by progressively shortening the duration of the activities in critical path method (CPM) network schedule, while monitoring the change in the total project cost.

PREVIOUS WORK ON SCHEDULE COMPRESSION

Several authors have proposed a number of varying techniques for reducing the overall duration of a construction project. Yau (1990) describes a heuristic method that is based on control actions and addresses the shortcomings of resource constrained projects, which have started to overrun budget. These control actions speed up the project and involve reallocating resources between activities and, if necessary, including the addition of extra resources. Therefore, this heuristic method, accelerates resource constrained projects, which, in turn, preserves the activity schedule.

Moselhi (1993) presented a new method for critical path scheduling that optimizes the project duration in order to minimize the total project cost. This proposed method is based on the well-known direct stiffness method for structural analysis and establishes a complete analogy between the structural analysis problem, with imposed support settlement, and project scheduling with an imposed target completion date.

Wu and Li (1996) investigated the problem of determining exactly where the compression effort should be placed so that the schedule can be improved more efficiently. They developed an evaluation methodology based on priority weight

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computations that not only indicates where the compression effort should be placed but the activities affected and the amount by which the job can be compressed.

Al-Tabtabai and Alex (1998) also presented a new method to compress a construction schedule network based on the application of genetic algorithms. From this study, indications were given on how genetic algorithms can contribute in solving construction-related optimization problems.

While these studies presented various techniques for performing schedule compression, it is important to note that they were all accomplished using deterministic values for the cost and duration of the project activities. However, since it is not possible to know the exact value of each activity's duration and cost, prior to the execution of these activities, these techniques cannot be applied with a high degree of confidence, because there is no absolute provable least-cost schedule solution that can be derived for real world projects prior to the execution of project activities.

ACTIVITY BASED COSTING SIMULATION

A relatively new approach, based on discrete event simulation, has been implemented to simultaneously perform range estimating with probabilistic scheduling in order to produce the probability distributions for the project cost estimate and schedule. This tool is called Activity Based Costing Simulation and was developed as part of a research study undertaken at Texas A&M University.

Activity Based Costing Simulation is implemented through a software package called ABC-Sim. This package has the added advantage over traditional stand-alone range estimating and probabilistic scheduling applications of being able to simultaneously perform range estimating and probabilistic scheduling for an appropriately modeled construction project. Combining these tools results in a range of cost estimate and project schedules values. These values can be appropriately modeled as probability distributions for both the project cost estimate and the project schedule values. Additionally, for each iteration of the simulation process, a project schedule value and its corresponding cost estimate value are also produced. ABC-Sim output can be summarized as a scatter diagram depicting the general relationship between time and cost (Fig. 1).

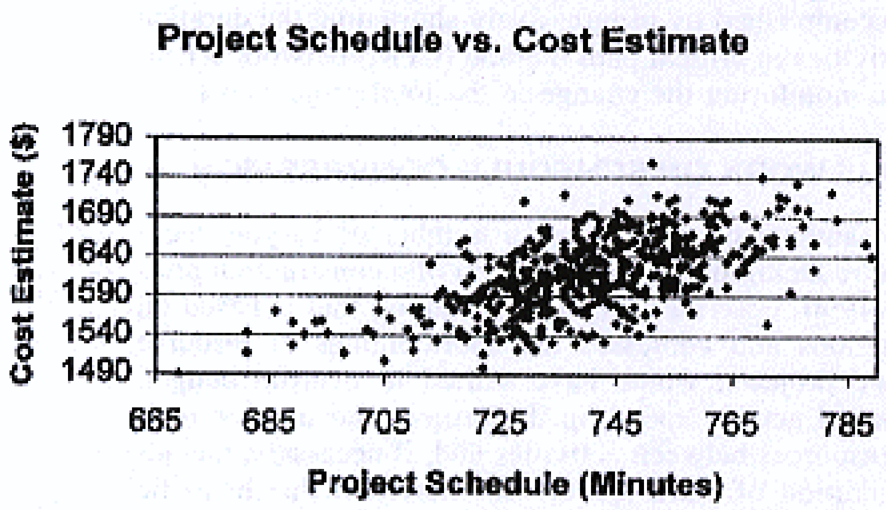

FIG. 1. Scatter Diagram of ABC-Sim Output Showing Project Schedule versus Cost Estimate

To make use of the information obtained from the simultaneous execution of range estimating and probabilistic scheduling for construction projects, it is necessary to find an adequate means of relating their respective data sets. Defining this relationship will make it possible to benefit from the fact that the schedule chosen to execute a project relates in some meaningful way to the cost estimate of that project. One reliable means of relating these data sets is through a process called the Multiple Simulation Analysis Technique (Isidore 1999). This technique provides a means of relating the cost estimate data for a specific project with its schedule data to simultaneously select high percentile values for both. It will then be possible to use this information in the planning of construction projects to minimize the risk involved with undertaking these projects.

LEAST-COST SCHEDULING APPLICATION

A sample construction operation (i.e., Project-E1), consisting of six simple activities, was developed to illustrate the application of least-cost scheduling. The schedule for this operation is as shown in Fig. 2 with the critical path (ACDF) and activity durations as indicated. A detailed breakdown of the respective activity duration and cost data for this project is presented in Table 1.

The normal duration represents the time that would typically be required to perform each activity under normal conditions with a sufficient amount of resources. The crash time represents the minimum amount of time during which it is also possible to complete each activity, but in this case, it requires additional resources to do so. The normal cost and crash cost are the costs associated with the normal and crash durations, respectively.

In this simple example, the total project cost is made up of two primary cost items—the direct cost and the overhead cost. Both the normal cost and crash cost are typically referred to as the direct cost and are defined as the cost of the labor and materials required to perform the work necessary to complete each activity. The total direct cost represents the cost of all labor, equipment, and materials required to complete the entire project. The overhead cost for Project-E1 is fixed at $100/day

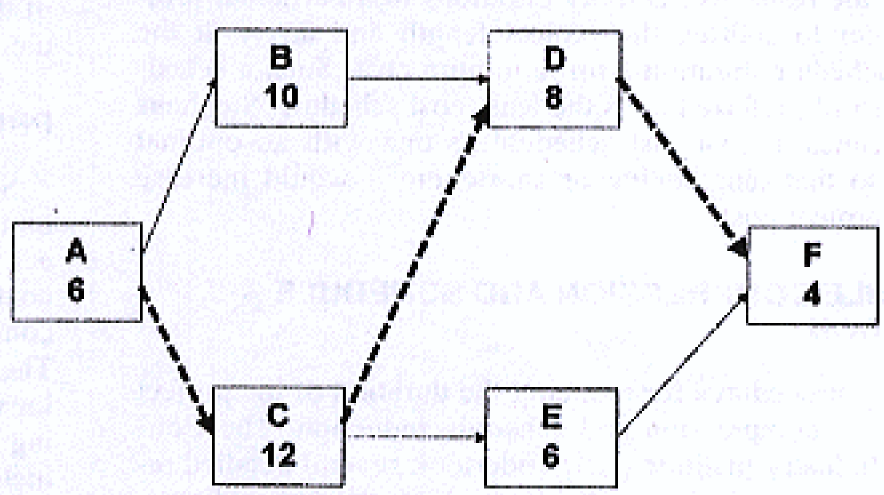

FIG. 2. CPM Schedule for Project-E1

TABLE 1. Activity Duration and Cost Data for Project-E1

	Durations		Cost (dollars)				
Activity	Normal	Crash	Normal	Crash 1	Crash 2	Crash 3	Crash 4
A	6	4	800	890	1,010	—	—
B	10	7	1,100	1,225	1,350	1,475	—
C	12	8	1,400	1,475	1,560	1,650	1,760
D	8	4	1,000	1,035	1,115	1,195	1,295
E	6	3	900	965	1,040	1,130	—
F	4	2	800	825	865	—	—

TABLE 2. Results of Least-Cost Schedule Exercise for Project-E1

Durations		Cost/Day Increase				Days shortened						
Activity	D-Days	Crash 1	Crash 2	Crash 3	Crash 4							
A	2	90	120									1
B	3	125	125	125								
C	3	74	85	90	110				1		1	
D	4	35	80	80	100		1			1		
E	3	65	75	90								
F	2	25	40			1		1				
		Days cut				1	1	1	1	1	1	1
		Project duration			30	29	28	27	26	25	24	23
		Increase $/day				25	35	40	75	80	85	90
		Direct cost			6,000	6,025	6,060	6,100	6,175	6,255	6,340	6,430
		Overhead cost			3,000	2,900	2,800	2,700	2,600	2,500	2,400	2,300
		Total cost			9,000	8,925	8,860	8,800	8,775	8,755	8,740	8,730

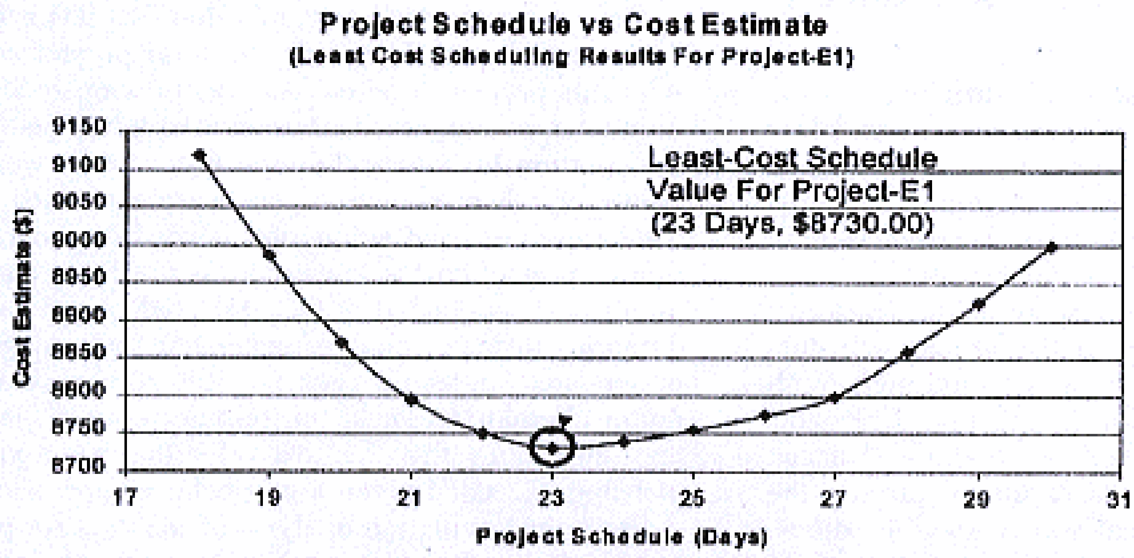

FIG. 3. Least-Cost Schedule Curve for Project-E1

and is multiplied by the project schedule to determine the total overhead cost for the project.

Based on the normal data provided for each activity, the critical path (ACDF) gives a project schedule of 30 days at a cost of $9,000. Performing a least-cost scheduling exercise for Project-E1 (Table 2) resulted in a least-cost schedule of 23 days at a cost of $8,730. This cost-duration data is plotted in Fig. 3, which illustrates that a minimum cost is attained at a project schedule of 23 days.

Once the least-cost schedule is attained, an examination of the project schedule shows that all paths through the schedule (ABDF, ACDF, ACEF) were critical. This indicates that the delay of any activity in the schedule during the execution of this project will void the least-cost schedule solution, result in a project duration longer than 23 days, and cost more than $8,730. Thus, trying to ensure that all activities are executed with the utmost efficiency to achieve the least-cost schedule becomes an important, yet considerable, task to perform.

LIMITATIONS OF LEAST-COST SCHEDULING

While it is not necessary for all the paths in a project to become critical during a least-cost scheduling exercise, there are several limitations with this procedure that reduce its effectiveness. According to Stevens (1990), there is no absolute, provable least-cost solution to a real-world project, because the activity durations and cost values cannot be known exactly in advance. This means that the use of deterministic values for activity cost and duration to produce a least-cost schedule is not a very reliable means of arriving at an optimal cost and schedule. It is, thus, hypothesized that the use of a probabilistic technique should provide more reliable results.

PROBABILISTIC COST-SCHEDULE MINIMIZATION

Having identified several crucial limitations of deterministic least-cost scheduling, a probabilistic approach was created to overcome these limitations. Using the data presented in Table 1, uniform probability distributions were assigned to the duration and cost for each activity as shown in Table 3. This distribution was chosen based on the assumption that the activity duration and cost were equally likely to be any value between the highest and lowest available data points. It should be noted that other probability distributions may be used in actual projects when these distributions better reflect the project uncertainties.

Using this data and the project schedule logic shown in Fig. 1, Project-E1 was again input into the ABC-Sim software with the appropriate duration and cost data assigned to each activity and simulated once using 500 runs. The results of this simulation run are summarized in Table 4 and show that, on average, the project schedule obtained using simulation is 24 days, and the total cost is $9,209.

When compared with the results obtained from the least-

TABLE 3. Probabilistic Input Data for ABC-Sim (Project-E1)

		Activity Duration		Activity Cost (dollars)	
Activity	Distribution type	L	H	L	H
A	Uniform	4	6	800	1,010
B	Uniform	7	10	1,100	1,350
C	Uniform	8	12	1,400	1,760
D	Uniform	4	8	1,000	1,295
E	Uniform	3	6	900	1,130
F	Uniform	2	4	800	865

TABLE 4. Comparison of Least-Cost Scheduling and Probabilistic Simulation Results for Project-E1

Factors	Schedule duration (days)	Total project cost (dollars)
Least cost scheduling	23	8,730
Average simulation values	24	9,209
Percent difference	0	6

cost scheduling exercise performed in the previous section, it was observed that the average project schedule duration obtained from the simulation analysis was approximately the same as the result obtained from the deterministic least-cost scheduling analysis. However, there was some difference in the cost value obtained from least-cost scheduling and that obtained from the simulation analysis.

ANALYSIS OF SIMULATION RESULTS–SCHEDULE DURATION

While there does not appear to be much difference in the average schedule value obtained from the stochastic simulation and that obtained from deterministic least-cost scheduling, it can be shown that these values are not equivalent. This difference stems primarily from their respective interpretation in addition to the way in which they were generated.

As was mentioned in the previous section, the least-cost schedule value of 23 days represented the project schedule duration at which the total project cost was a minimum. With respect to the probabilistic simulation output, Fig. 4 presents a frequency plot of the possible schedule durations that can reasonably be expected to occur as a result of varying the activity duration between their normal and crashed durations. This plot clearly indicates that the schedule duration of 24 days is the project duration with the highest likelihood of occurring, given the sequence of activities shown in Fig. 2 and the historical activity data in Table 1.

Fig. 4 shows that a 24-day project schedule has a relative frequency of 22.4% (or a 22.4% chance of occurring), while a schedule duration of 23 days has a relative frequency of 22% (or a 22% chance of occurring). Since these percentage values were not considered to be significantly different, a choice of either 23 or 24 days would be acceptable as the compressed schedule duration for Project-E1; a final selection between the two would depend primarily on the availability of additional project related information. All other possible schedule durations were shown to have a smaller relative frequency; selection of these would considerably reduce the chances of being able to complete Project-E1 during those respective times.

Thus, based on the stochastic duration output, it would be very likely that selecting the least-cost schedule duration of 23 days would have provided the contractor with a 22% likelihood of being able to complete the project during this time. Having access to this information beforehand can potentially have serious cost implications for both an owner and a contractor. If, however, a compressed project duration of 24 days is selected, there is still a 22.4% chance that the project can actually be completed using this schedule duration. The likelihood of actually meeting this compressed schedule duration is also considerably higher than any of the other possible compressed schedule durations. Overall, choosing a project duration of 24 days would have a 44% chance of occurring if planning is concentrated on meeting the 23 days compressed schedule, which already had a 22% chance of occurring.

ANALYSIS OF SIMULATION RESULTS—TOTAL PROJECT COST

Once a compressed schedule duration has been selected, it was necessary to determine the total project cost associated with this project duration. As was previously shown, the traditional application of least-cost scheduling resulted in a project duration for which the total project cost was a minimum. However, when the activity costs are expressed as a probability distribution, and simulation is used to generate a distribution of project cost values, there is no direct equivalent, minimum cost associated with a selected compressed scheduled duration. Instead, what is generated, is a range of possible project cost values for each possible compressed schedule duration alternative. Thus, the primary concern is not with the least cost, but rather the cost value that has a low probability of being exceeded given a particular compression scenario.

From the simulation analysis of the data for project-E1, directly relating the project schedule data to their corresponding cost estimates resulted in the plot shown in Fig. 5.

From this it can clearly be seen that at each compressed project duration alternative, there is a set of possible cost estimate alternatives. Also, there is considerable overlap of the cost estimate values signifying that the selection of a particular project schedule duration does not have mutually exclusive cost estimate values. Thus, a particular cost estimate value may be acceptable for a range of possible project durations and invalidates the concept of a least-cost schedule. If, for example, a cost estimate of $9,000 is considered, Fig. 5 shows that this estimate is valid for a project duration ranging from 21 to 26 days. Using a schedule duration of 22 days, the cost estimate value of $9,000 has an 49% chance of being exceeded (it represents the 51st percentile level cost estimate). However, for the schedule duration of 26 days, the same value ($9,000) represents the 16th percentile level cost estimate value and has

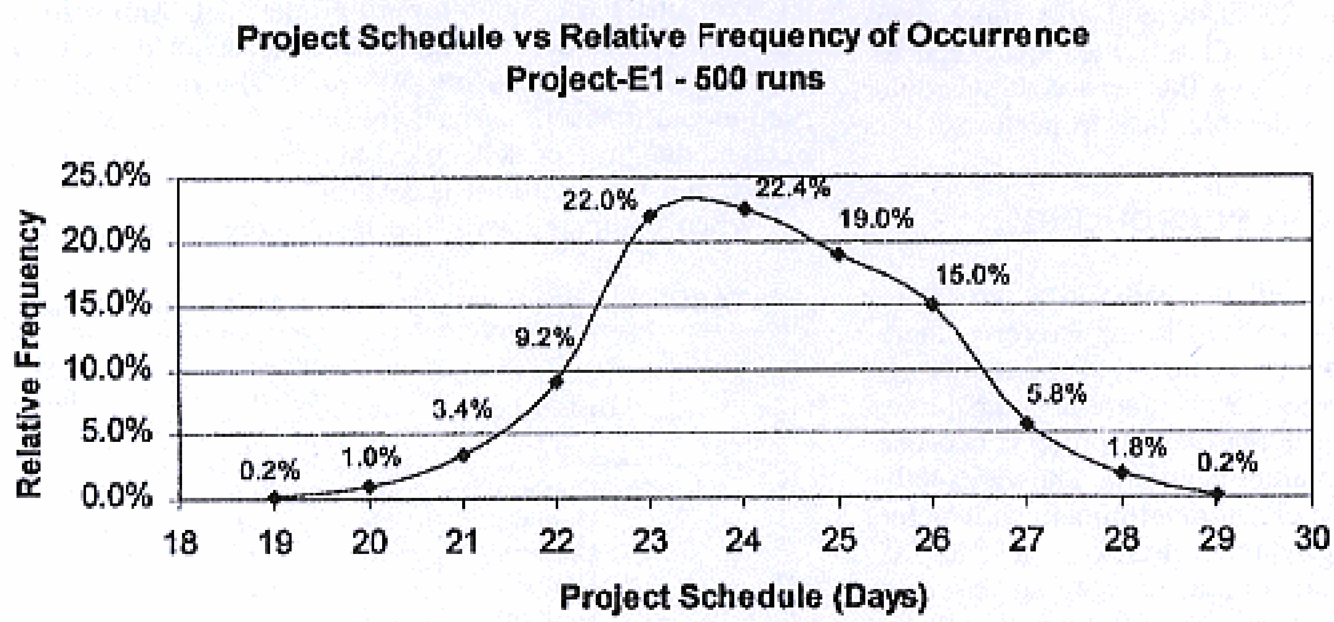

FIG. 4. Schedule Duration Frequency from 500 Simulation Runs

Project Schedule vs Total Project Cost
(Project-E1)

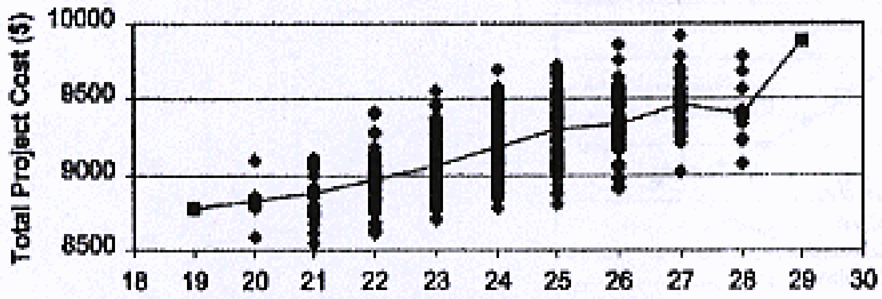

FIG. 5. Project Schedule versus Total Cost for Project-E1 Simulation Output Data

an 84% chance of being exceeded. Hence, while this cost estimate value is feasible for several possible compression scenarios, the confidence level associated with it varies considerably depending on which compressed duration is considered.

Considering the example deterministic least-cost value of $8,730, determined as the least-cost schedule of 23 days, examination of Fig. 5 indicates that this value is possible for a compressed duration of 21, 22, and 23 days. In all cases, however, this cost estimate value had a low confidence level, and there was a very high probability of it being exceeded. Thus, if this deterministic cost estimate value had actually been used for a compressed duration of 23 days, it would represent the 3rd percentile level, and there would be a 97% chance of it being exceeded. Hence, it would have been very unlikely that the project could have been successfully completed for this amount. The important consideration, therefore, becomes choosing a cost estimate that directly relates to the project duration of interest, at a sufficiently high percentile level, such that, there is a very small chance of exceeding that value.

One way of arriving at an appropriate cost estimate value is through the application of a new technique called "Multiple Simulation Analysis Technique or MSAT" (Isidore 1999). This technique is used to relate the results of range estimating and probabilistic scheduling such that high confidence level values can be selected for a project cost estimate and schedule. It not only allows for the simultaneous selection of the cost estimate and schedule, but allows these values to be related in some meaningful way. If the most likely compressed schedule duration of 24 days is considered, for example, then the application of MSAT involves the following steps:

1. Select the project schedule value of interest. In this particular case, the compressed duration of 24 days is selected as the schedule duration of interest.
2. Use the simulation model in ABC-Sim to simultaneously perform range estimating and probabilistic scheduling to generate the data shown in Fig. 5.
3. From this dataset, select the schedule values having a duration of 24 days along with their corresponding cost estimate values.
4. Determine the percentile level of the resulting cost estimate values by sorting the simulation output. For the data set shown in Fig. 5, a partial listing of the cost estimates and their associated confidence level for a schedule duration of 24 days is shown in Table 5.

The cost estimate values and their associated percentile level, derived from the procedure described above, were fitted with (1), which represents a first-order linear regression equation. This regression relationship is shown graphically superimposed on the scatter diagram of the simulation output data presented in Fig. 6. The R^2 value of 0.98 shown on Fig. 6 indicated that using the confidence level helps to explain approximately 98% of the variability in the cost estimate, when specifically considered at a project duration of 24 days

$$CE_i = 689.68PL_i + 8855.7 \quad (1 \geq PL \geq 0) \qquad (1)$$

Using (1), it is possible to predict a cost estimate value that has a high percentile level (or a low probability of being exceeded) for project-E1. For example, if the 90th percentile level cost estimate value was desired (the cost estimate with a 10% chance of being exceeded), (1) produced a project cost estimate of $9,505. The minimum value from this data set for the project duration of 24 days is $8,778 while the maximum is $9,700. Hence, while the 90% confidence level does not represent a minimum analogous to the least-cost schedule value of $8,730 for a 23 days compressed schedule, the use of MSAT allowed a quantifiable percentile level to be associated with the selection of the project cost estimate. Addi-

TABLE 5. Partial Listing of Project-E1 Simulation Output Data

Compressed project schedule (days)	Project cost estimate (dollars)	Cost estimate percentile ranking
24	9,284	0.64
24	9,013	0.18
24	9,603	0.97
24	9,156	0.41
24	8,822	0.03
24	8,916	0.08
24	9,109	0.32
24	9,301	0.67
24	9,298	0.67
24	9,357	0.76
Average standard deviation	9,209 251	0.508 0.302

Percentile Level vs. Cost Estimate
(Compressed Project Schedule = 24 Days)

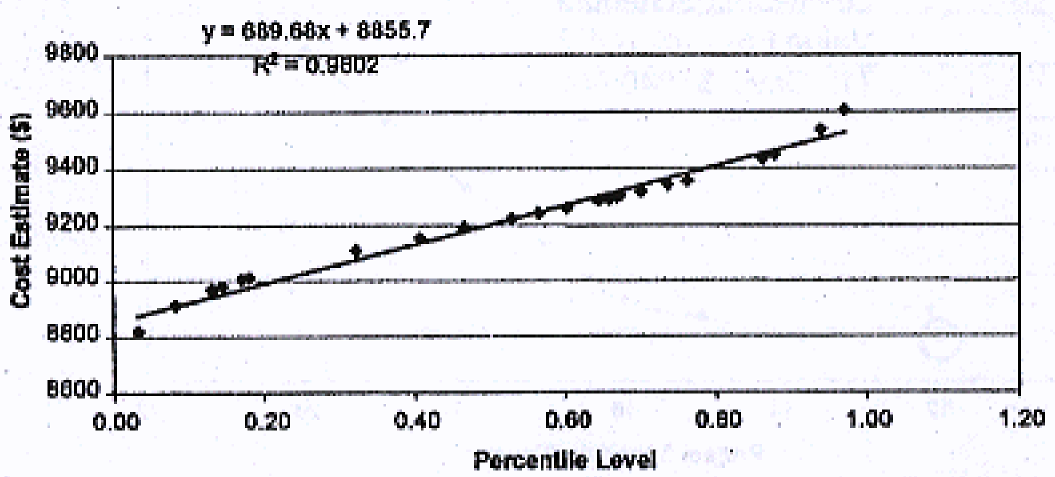

FIG. 6. Linear Regression of Percentile Level versus Cost Estimate for Project-E1

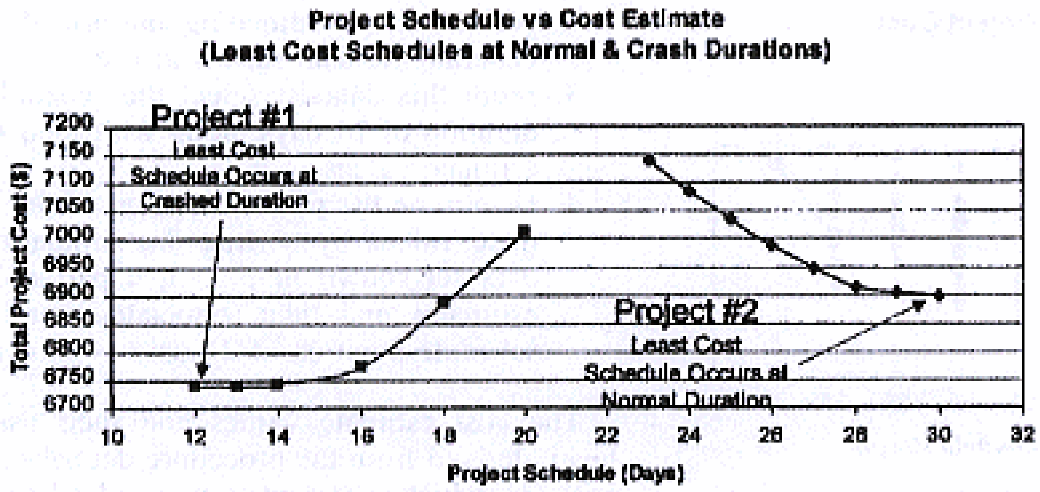

FIG. 7. Alternative Least-Cost Schedule Solutions

tionally, it provides some way of allowing various percentile level, cost estimate alternatives to be quickly determined. Since the cost estimate obtained using the probabilistic approach represents an optimal rather than a minimum value, the entire procedure is perhaps best described as Probabilistic Optimal-Cost Scheduling (POCS).

ALTERNATIVE LEAST-COST SCHEDULES

Project-E1, considered in the previous section, was shown to have a least-cost schedule that occurred at some intermediate point between the normal duration and the maximum crashed duration possible. This optimal point occurred at a project duration of 23 days and graphically was shown to resemble a parabola with its vertex at 23 days (Fig. 3). There are, however, some instances where the least-cost schedule occurs at either the normal duration or at a crashed project duration, as illustrated in Fig. 7.

When the least-cost schedule for Project-E1 was compared with the average value obtained using the probabilistic method of determining an optimal crashed project duration, it was demonstrated that the values predicted by both methods were approximately equal. However, there are some instances where this is not the case. If another project is considered, Project-E2, represented by the schedule network shown in Fig. 8, a least-cost scheduling analysis can be carried out using the project data shown in Table 6.

Using a fixed overhead cost of $90/day, the solution to the deterministic least-cost schedule exercise for this project is presented in Fig. 9. From this data, it is possible to see that the least-cost schedule for this project occurs at a project duration of 12 days with a cost of $7,940.

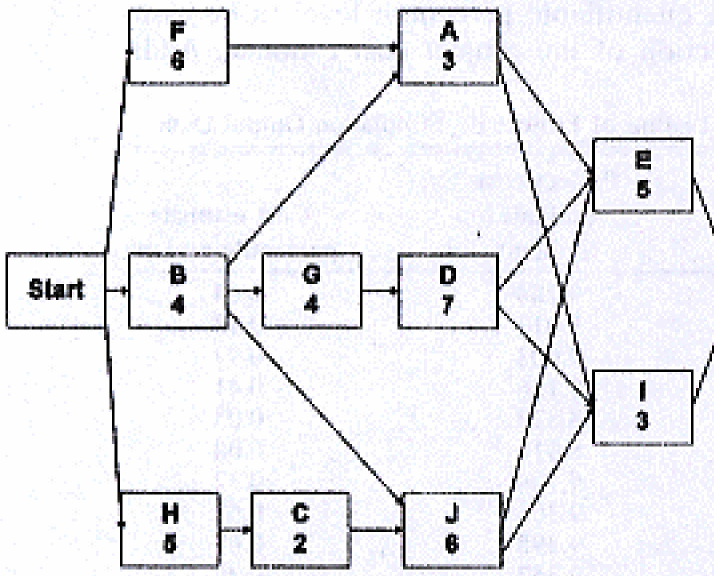

FIG. 8. CPM Schedule Network for Project-E2

TABLE 6. Activity Duration and Cost Data for Project-E2

	Durations		Cost (dollars)	
Activity	Normal	Crash	Normal	Crash
A	3	2	400	430
B	4	2	1,020	1,100
C	2	2	350	350
D	7	5	1,250	1,300
E	5	3	825	895
F	6	3	610	700
G	4	2	430	490
H	5	2	525	660
I	3	2	390	410
J	6	3	615	765

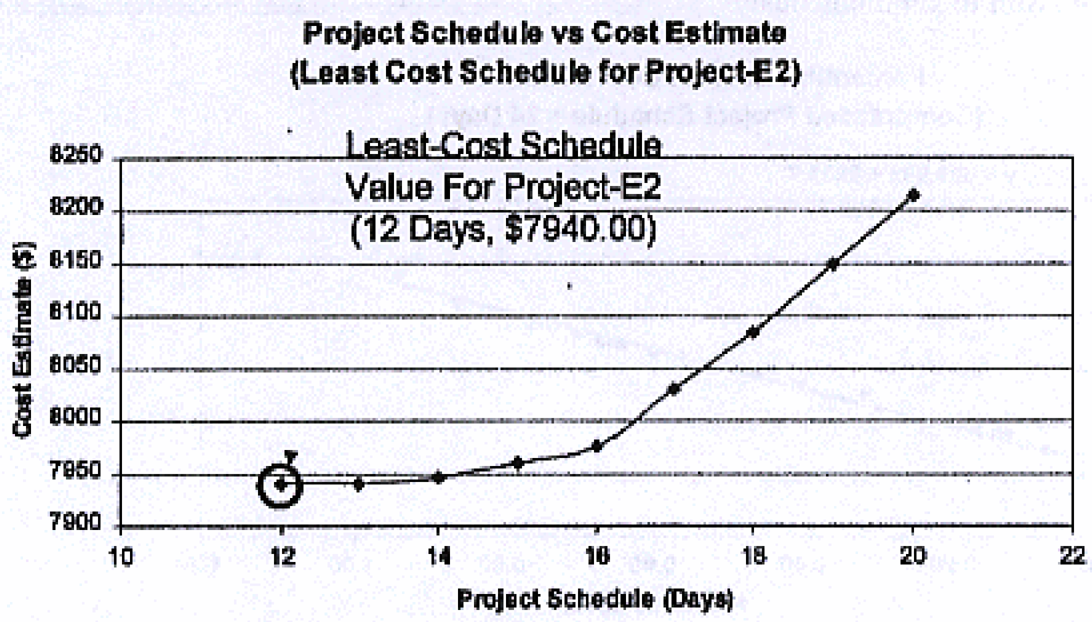

FIG. 9. Least-Cost Schedule Curve for Project-E2

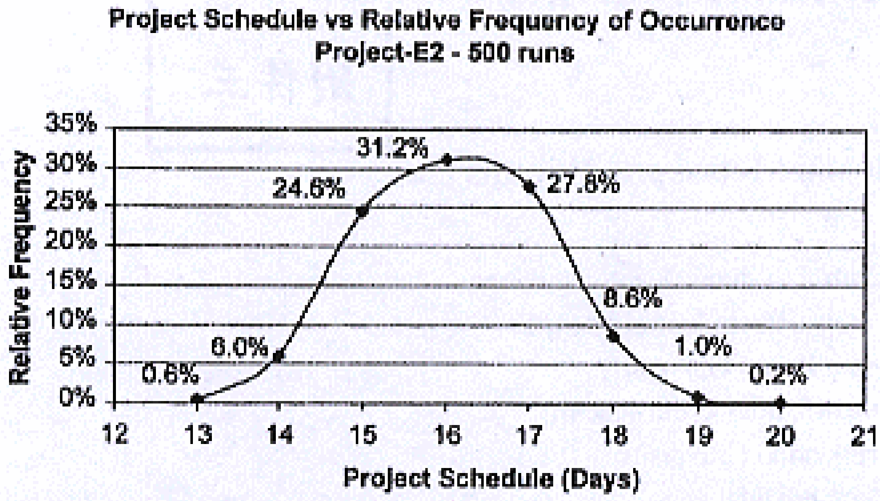

FIG. 10. Schedule Duration Frequency Based on 500 Simulation Runs

Probabilistic Approach for Alternative Least-Cost Schedules

After determining a deterministic least-cost schedule solution for Project-E2, the next step involved performing a probabilistic optimal-cost scheduling exercise. As was previously presented, the network logic was entered into ABC-Sim with the appropriate duration and cost data and then simulated once using 500 runs. The duration and cost data were again assigned a uniform distribution using the assumption that any value within these ranges could occur with equal likelihood.

The results of the simulation analysis are graphically summarized in Fig. 10. This figure shows that the most likely compressed project duration is 16 days which is significantly different from the least-cost schedule result of 12 days. This difference is attributed to the fact that the probabilistic approach provided the schedule duration that is most likely to occur given the particular sequence of activities in the schedule network, and the variability inherent in the duration data. Thus, the probabilistic approach does not attempt to mimic the least-cost schedule approach, but instead provides a more realistic expectation of what is likely to happen given that it is possible to repeatedly execute Project-E2 using discrete event simulation. The process of obtaining the project cost estimate for the compressed, probabilistic schedule duration is carried out in a manner similar to that presented for Project-E1.

ADVANTAGES OF PROBABILISTIC OPTIMAL-COST SCHEDULING (POCS)

Using a probabilistic approach to arrive at a compressed project duration not only makes better use of the available historical data, but it also provides additional information to facilitate the planning process associated with the reduction of a project schedule. As was previously stated, since there is no absolute, provable, least-cost solution for a real project, a least-cost schedule solution cannot be applied with any high degree of confidence and will be very difficult to achieve in practice. The primary difficulty with achieving such values is the inability to deterministically anticipate what the impact and outcome of least-cost scheduling will be on the project.

Being able to determine which compressed project duration is most likely to occur, and the frequency of the possible alternatives, enable detailed planning to be undertaken for this schedule option. Additionally, it allows contingency planning in the event that one of the other compression project durations or critical paths is realized. This significantly reduces the element of surprise during project execution and greatly increases the chances of successfully completing the project in the reduced time specified.

CONCLUSIONS

Based on the analysis, it was shown that the application of the deterministic technique of least-cost scheduling to arrive at the shortest possible schedule duration at the minimum cost is a somewhat unreliable means of compressing a project schedule. This unreliability stems from the fact that there is no absolute, provable, least-cost schedule, because the actual values for the activities making up a project cannot be known exactly until those activities are completed in the field.

Thus, it was necessary to develop a new, stochastic technique to more reliably determine a compressed schedule duration and its associated cost estimate for which the percentile level could be quantified. This new technique referred to as POCS used discrete event simulation to simultaneously perform range estimating and probabilistic scheduling using the historical data available for each respective activity. These results are then analyzed and related, in a statistically significant way, to arrive at a compressed schedule duration and cost estimate that can both be generated and applied with a higher degree of confidence than was possible without the availability of this technique.

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Client Satisfaction and New Direction of Architects' Services

附件三

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Abstract

This paper is a report on research that is part of a strategic study for the Japan Institute of Architects (JIA) on the relationship between architects and their clients. In this paper, four research results are presented. The first is a structural modeling of the relationship between the clients' satisfaction and architects' services. The second is an analysis of architects classified into two groups; design-oriented architects and management-oriented architects, based on their actual services. The third is a review of the JIA's services standard, and the fourth is an assessment of their services and strategies based on the Self-Diagnosis Sheet developed in this study.

Keywords: customer satisfaction; architect; design services; client; self-diagnosis

Introduction

This paper reports on research that is a part of a strategic study for the Japan Institute of Architects (JIA) regarding architects and their clients. The background is as follows. Because of the variety client of needs, there are gaps between clients' satisfaction and architects' services. As a result, the number of clients who are not satisfied with their projects is increasing. How architects cope with this situation, is in fact one of our most important problems. The purpose of this study is to grasp the present situation and take measures to improve it. The authors carried out this study from 1995 to 1999 based on the six steps below (Furusaka, 1998a; Furusaka, 1998b; Miisho, 1999a; Miisho, 1999b and Kaneta, 1999).

Step 1: Customer satisfaction (CS) research of long-term clients.
Step 2: CS research of general clients.
Step 3: Classification of architects' by type according to their actual services and new directions.
Step 4: Investigation measures to match architects' services to the client's needs.
Step 5: Proposal concerning the architects' role in building production systems.
Step 6: Proposal of the Self-Diagnosis Sheet to assess architects' services and their strategies.

In the American Institute of Architects (AIA) and the Royal Institute of British Architects (RIBA), investigation research to understand what clients need was carried out in order to construct the vision of the architect of the 21st century and the professional group of the future (AIA, 1993; RIBA, 1993). How satisfied the clients are towards the architect is investigated in Japan while the construction industry of Japan has changed along with the diversification of building activity (Mitsubishi, 1990). This research and its results more clearly elucidate the form of this disparity that exists between the architect's services and the clients' satisfaction.

CS of Clients

The difference in social conditions or clients' knowledge and experience causes a disparity in the clients' expectations and evaluations of the architects' services. Adopting the concept of CS, the authors define the architects' services, with which clients are satisfied or dissatisfied regarding their projects.
The degree of CS is defined by the degree of the gap between the client's expectations and objective evaluations (Fig.1). Services delivered by architects are classified as follows:

- "Expected satisfaction" services: high expectations and high evaluations.

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- "Unexpected satisfaction" services: low expectations and high evaluations.
- "Non-satisfaction" services: low expectations and low evaluations.
- "Dissatisfaction" services: high expectations and low evaluations.

Managing "unexpected satisfaction" and "dissatisfaction" services is given high priority.

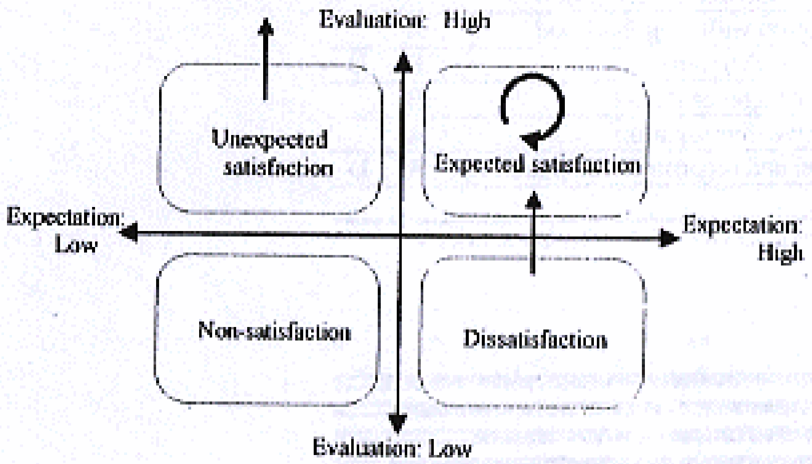

Fig. 1. The concept of CS
(Source: Furusaka, 1998a)

The research was carried out on clients from June to September 1995, and was carried out based on a questionnaire format. The number of valid answers was 98. (Furusaka 1998a) The services were extracted from JIA's "Standard Form of Architect's Services" in 1992 referring to three architects' suggestions. According to the concept of CS, "unexpected satisfaction" and "dissatisfaction" services were extracted (Fig.2).

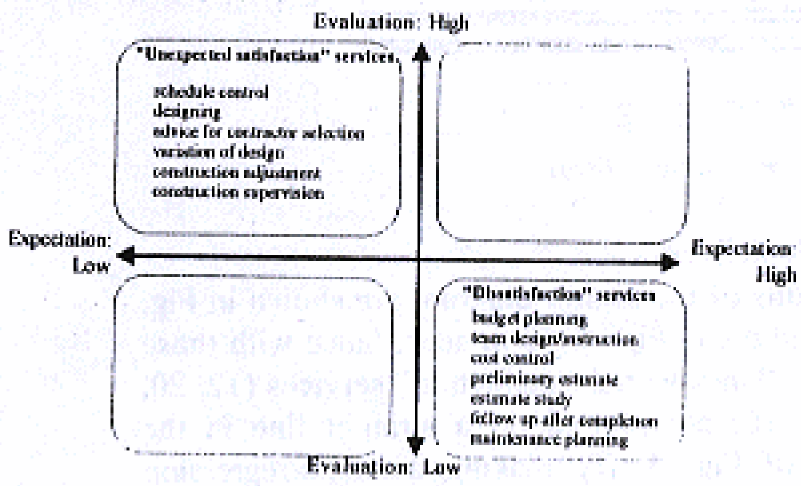

Fig. 2. "Unexpected satisfaction" and "dissatisfaction" services
(source: Furusaka, 1998a)

The findings are summarized as follows:

- "Unexpected satisfaction" services are "schedule control", "designing", "advice for contractor selection", "variation of design", "construction adjustment" and "construction supervision".
- "Dissatisfaction" services are "budget planning", "team design/instruction", "cost control", "preliminary estimate", "estimate study", "follow up after construction", and "maintenance planning".
- These "dissatisfaction" services exist in cost planning at the early stage of the project, cost control during the project, in follow-up services and in maintainability design.
- These dissatisfactions can be part of what brings about the problematic gap in the relationship between clients and architects.

Actual Services of Architects and Their New Direction in the Future

As mentioned above, "dissatisfaction" services were shown. It is demanded that architects not only deliver without fail the high standard services expected by clients, but also that they deal intensively with services of which there is a low expectation, beforehand. In order to do so, it is important that they show their high quality services, that the clients are able to select architects who can deliver to their expectations, and that selected architects take the responsibility for delivering them. In this chapter, the actual conditions of delivery of services by architects are defined.

Basic Services of Architects

The research was carried out in August 1996 on architectural firms that are members of the JIA. They numbered 140. The authors define less than four member architect firms as smaller firms, and more than 100 as larger firms.

According to the JIA's Standard, there is a distinction between basic services, which are always delivered, and additional services, which are delivered based on the needs of clients (Table 1). However, there are also basic services which are delivered secondarily. These are "team design", "budget planning" and "schedule planning" at the early stage of the project; and "guarantee against defects" and "maintenance management" at the late stage of the project (Fig. 3). Regarding the above services, there is often a serious lack of information.

New Direction of Architects

The services in which architects have the possibility to expand their scope in the future could include the following: "study/survey" "budget planning" "outline proposal" "outline planning" "preliminary estimate" in the early stage of the project. This possibility is seen among additional services especially.

Structural Modeling of Architects' Services and Clients' Satisfaction

A structural modeling between the architects' services and the clients' satisfaction is made based on the following assumptions:

- It is possible for the clients' satisfaction to represent the characteristics of whole firms from the research data carried out on each client concerning their project.

Table 1. Architects' Services

Architects' Services			Architects' Services'		
1 Team design	B	M	14 Familiarity with laws and ordinances	B	
2 Budget planning	B	M	15 Explanation for third party	A	
3 Schedule planning	B	M	16 Construction contract	B	M
4 Outline proposals	A	D	17 Construction cost accounting	B	
5 Study / survey	A	D	18 Alternative proposal	A	
6 Outline planning	A	D	19 Construction adjustment	B	
7 Preliminary estimate	A	M	20 Confirmation and investigation	B	
8 Project intentions	B	D	21 Negotiation with neighborhood	A	
9 Scheme design	B	D	22 Advice for maintenance	B	D
10 Documentation (sheme design)	B	D	23 Guarantee against defects	B	
11 Construction cost (scheme design)	B	M	24 Successive investigation	A	
12 Detailed design	B	M	25 Addition and reconstruction planning	A	D
13 Construction cost (detailed design)	B	M			

B: Basic services / A: Additional services / M: Management services / D: Design services

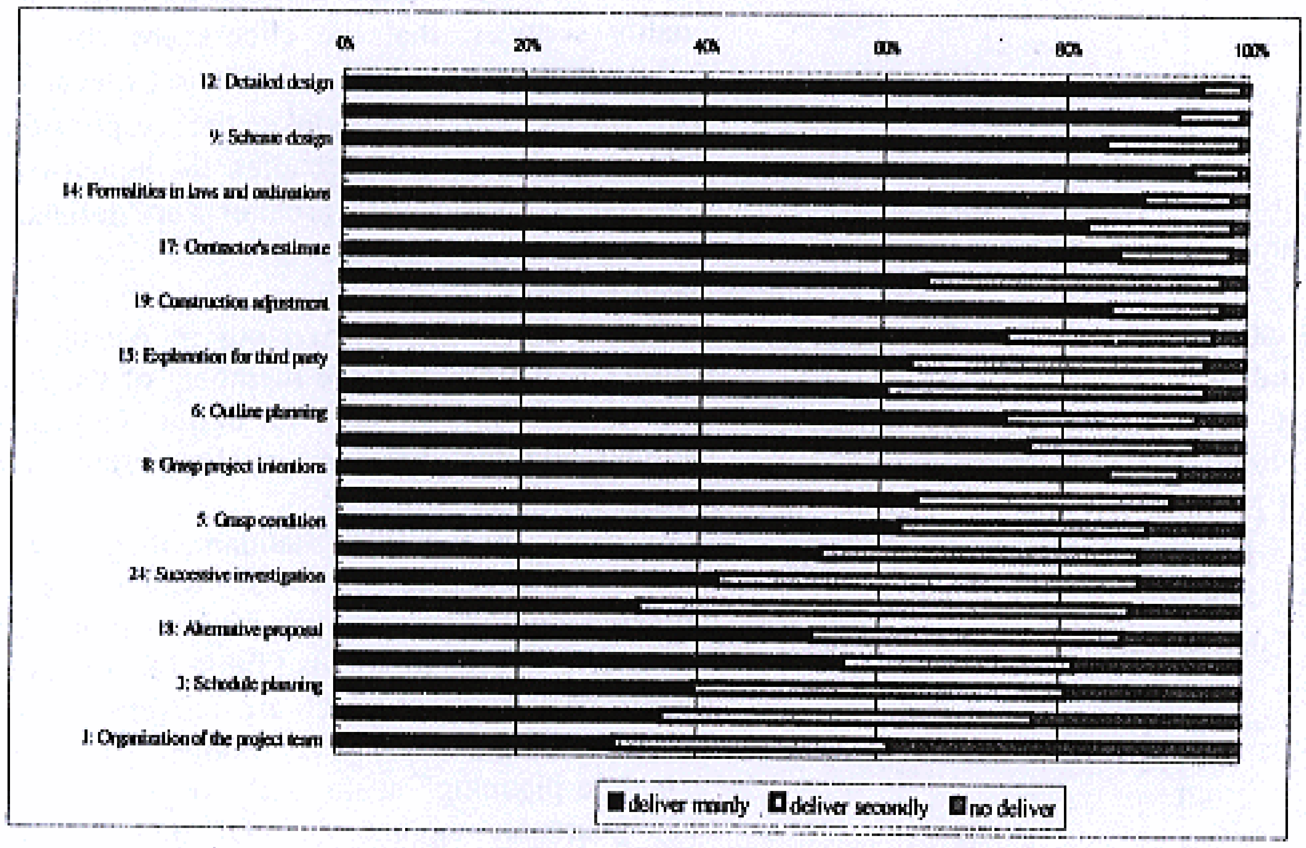

Fig. 3. The Condition of Dlivering Services (source: Miisho, 1999a)

- It is equally possible to define the clients' direction.
- It is possible to evaluate the service quality based on the number of delivering firms.

Structural modeling of the gap between the architects' services and clients' satisfaction

"Unexpected satisfaction" services and "dissatisfaction" services are defined in Table 2. A structural modeling of "the ratio of the firms delivering the service" is made. That is, the sum of the ratio of the firms delivering primarily and secondary with relation to the clients' satisfaction. The modeling is carried out according to the following hypotheses:

- JIA, as with all architectural firms, do not deliver in the same ratio for each service.
- There is a difference between services delivered primarily and secondarily. For example, if the ratios of the firms delivering two different services are the same, clients place a higher evaluation on the service that firm mainly delivers.

The results of the scatter diagram are shown in Fig. 4. The numbers in Fig. 4 are in accordance with those in Table 2. "Unexpected satisfaction" services (12, 20, 19, 9 and 16) are plotted on a straight line in the upper-left of Fig. 4. By making a single-regression analysis about them and inserting that result, a resulting straight line is gained - the architects' delivery line. This indicates the direction of service delivery of the JIA.

Architects' delivery line is defined by the equation as follows:

$$aJmain + bJsub = c \qquad (1)$$

where $a = 1$, $b = 1.07$, $c = 99.34$.

$$0 \leqq Jmain \leqq 99.34,\ 0 \leqq Jsub \leqq 92.84 \qquad (2)$$

where

Jmain: the ratio of the firms delivering primarily

Jsub: the ratio of the firms delivering secondary

Table 2. Delivery of Architects' Services

Evaluation	CS Research		Architects' Services		Delivering firms
Unexpected satisfaction	Schedule control	→	3	Schedule planning	Less
	Designing	→	9	Scheme design	More
	Designing/Variation of design	→	12	Detailed design	More
	Advice for constructor selection	→	16	Construction contract	More
	Construction adjustment	→	19	Construction adjustment	More
	Construction supervision	→	20	Confirmation and investigation	More
Dis-satisfaction	Estimate study	→	17	Construction cost accounting	More
	Cost control	→	13	Construction cost (detailed design)	Diversity
	Preliminary estimate	→	11	Construction cost (scheme design)	Diversity
	Follow up after completion	→	25	Addition and reconstruction planning	Diversity
	Maintenance planning	→	22	Advice for maintenance	Diversity
	Follow up after completion	→	23	Guarantee against defects	Less
	Cost control	→	7	Preliminary estimate	Less
	Maintenance planning	→	24	Successive investigation	Less
	Budget planning	→	2	Budget planning	Less
	Team design/instruction	→	1	Organization of the project team	Less

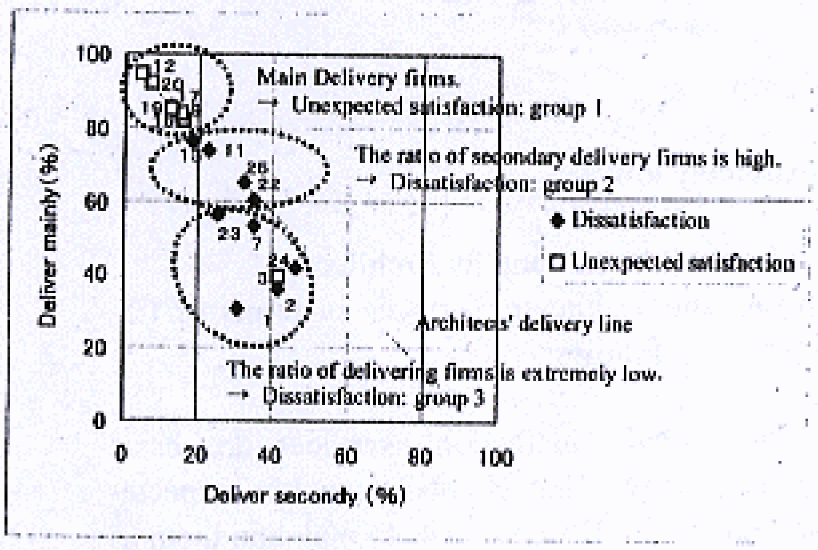

Fig. 4. Distribution of "Unexpected Satisfaction" and "Dissatisfaction" Services

The regression is carried out under unstable conditions. Therefore the authors give the value of "c" as +/-5 points. "Dissatisfaction" services do not meet the condition of the equation below:

$$f = aJmain + bJsub \qquad (3)$$

where $f \geqq c,\ a = 1,\ b = 1.07,\ c = 94.34$

The ratios for [23, 7, 24, 2, 1] do not meet the condition of (3). The low the number of delivering firms causes the clients' "dissatisfaction". In the services which met the conditions in (3), it is expected that the difference between "unexpected satisfaction" services and "dissatisfaction" services can be connected with the ratio of the firms delivering primarily or secondary. The cutoff point that classifies these services into two is the ratio, 20%. "Unexpected satisfaction" services and "dissatisfaction" services are divided at this cutoff point. To summarize so far:

- The direction of delivery services of the JIA for each firm is defined as the Architects' delivery line according to the equations (1) and (2).
- The services dealt with in the analysis are made into a structural model made up of three groups.
  Group 1: (Unexpected satisfaction) Services meeting the conditions of (1) to (3).
  Group 2: Dissatisfaction) Services with a high ratio of firms delivering secondary.
  Group 3: (Dissatisfaction) Services missing from the number of firms delivering.

Architects Classification According to Their Own Perceptions

Architectural firms can be classified according to the difference in their degree of emphasis on design content. A structural model can then be made between each type of architectural firm and the clients' satisfaction. Steps of the classification are as follows:

- Arrange 11 design contents characterizing firms in Table 3 from the characteristic content from design factors to management factors, and assign them standardized points.
- Seek architectural firms opinions on comparisons and ask them to assign the content an evaluation degree from 1-5.
- Based on the above data, calculate the degree of emphasis on each content in the design process using the Analytic Hierarchy Process.
- According to the consistency index, evaluate the consistency of architects' answers. There are 66 firms who gave valid answers.
- Multiply the standard points of each content by the degree of the emphasis, and then, sum the total. This value is the score for the index of design/management.
- Divide into three categories between the high scores and the low scores. The three categories are named Design-Oriented Group, Moderate

Table 3. Design Contents

Rank	Contents	Standard point	Rank	Contents	Standard point
1	Design	100	7	Advice for maintenance	40
2	Consulting at early stage	90	8	Neighborhood	30
3	Environmental symbiosis	80	9	Formula of procurement	20
4	Safety	70	10	Cost planning	10
5	Practicality	60	11	Schedule control	0
6	Constructability	50			

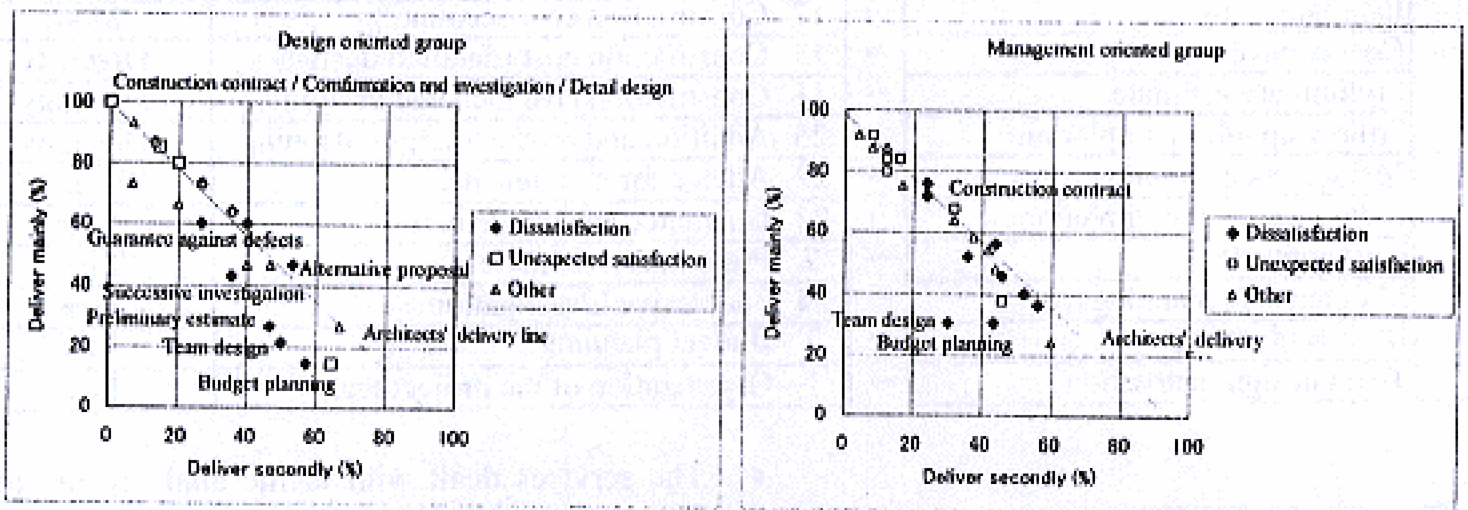

Fig. 5. Distribution in each group

Orientated Group, and Management-Oriented Group.

Based on the above steps, the 66 firms were classified with 15 firms as Design Oriented Group, and 25 firms as Management Oriented Group. The results of the scatter diagram of the services are shown by group in Fig. 5. The characteristics of each are as follows:

Design-oriented Group

- The ratio of the firms delivering "unexpected satisfaction" services is high. This is especially so with nearly all the firms delivering "construction contract" "investigation" and "detail design".
- The number of the firms delivering services with a low delivery ratio is minimal, but larger than that in the Management-oriented group. These are "team design" "budget planning", "preliminary estimates" and "guarantee against defects".

Management-oriented Group

- The ratio of the firms delivering "dissatisfaction" services is high compared with the Design-oriented group.
- The number of the additional services that meet the conditions of (2) is larger than that in the Design-oriented group.

To summarize so far:

- In the Design-oriented group, there are differences in the number of firms according to their services. Many firms deliver the services that they have the primary capability of carrying out (good field), but on the other hand there are many services of which the ratio of the firms delivering is extremely low.
- In the Management-oriented group, there are few services of which the ratio of the firms delivering is extremely low.

The Problems of the JIA and its Architects

Analyzing the architects' services in terms of CS, the results are as follows:

- Basic Services

All "unexpected satisfaction" services are basic services. This shows that clients' have low expectations for them. If the JIA continues to maintain them as basic services, it is important that the client understands that the architectural firms primarily deliver the basic services and that clients then have a standard reliance and expectation regarding the firms. For the basic services of which the ratio of delivering firms does not meet the conditions, it is important to classify the architects into the Design-oriented group and the Management-oriented group, and to make the differences of basic services that are applied to each group.

- Additional Services

There are two strategies regarding the additional services for which clients' expectations are high. First, with respect to changing the additional services which are expected to be high, to basic services, the JIA can change the perception of architects, thus multiplying the number of firms delivering. Second, the JIA can continue to maintain them as additional services, and each firm can provide a high quality of delivery.

- New Architectural Services

There is no "unexpected satisfaction" among additional services. It is important to create new additional services in high potential demand areas.

Self-Diagnosis Sheet

The Purpose of the Self-Diagnosis Sheet

The purpose of the Self-Diagnosis Sheet is follows:

- To ascertain the position of each firm within the firms in the JIA
- To confirm the direction of service areas to provide in the future
- To ascertain definite goals in specific service areas
- To verify definite strategies to be developed

The Composition of the Self-Diagnosis Sheet

As has become already obvious, the service in which the client is dissatisfied is that deemed necessary at the beginning of the project process and also that after its completion. This specifically refers to the services regarding cost, management of the project team, and the aftercare carried out after completion. As shown above, aiming towards unifying firms design and management goals, and employing different services standards for each aim are seen to be effective.

In addition, when the firm carries out a Self-Diagnosis, it is appropriate to do so from the perspective of both the projects process in business areas provided, (Early stage, Middle stage, Latter stage), and the type of services (design services/ management services). Thus, 50 areas of service, including new services, which the client could see as worrying, have been organized into the Self-Diagnosis Sheet from 1-10 in each of the major categories of 1-5. (Table 4.)

1. Design services at the early stage (the creation of Requirement For Proposal (RFP)/ business plan/ future facility plan).

Table 4. The Structure of the Self-Diagnosis Sheet

1 Design services at the Early stage	1. Requirement for proposal	1	Checklist to ascertain user needs
		2	Strengthening of briefing/programming
		3	Checklist to understand laws and regulations
		4	Checklist to ascertain other environmental and social conditions
		5	An manual for the creation of Requirement For Proposal
	2. Business plan Future facility plan	1	Business plan consultation
		2	Finance plan consultation
		3	Facility management consultation, including future facility plans
		4	Procedure book for setting building grades
		5	Creation of planning proposal report
2 Design services at the Middle stage	3. Design and draft	1	Documented consensus of design content with client
		2	Presentation technique towards client
		3	Documented consensus with contractor on design concept
		4	Strengthening of CAD in creation of drawings
		5	Strengthening of database of design information
	4. Design quality assurance	1	Enforcement within range of supervising contract
		2	Preserving this companies quality assurance system
		3	Obtain ISO9000s certification
		4	Take out liability insurance
		5	Preserve performance evaluation and report system
3 Management services at the Early stage	5. Management of project team	1	Effective organization of all members necessary to a project
		2	Selection of other partners in design team (structure, equipment, etc.)
		3	Selection of contractor
		4	Organization and participation in design-build contract
		5	Providing to client an alternative project organization plan
	6. Planning and control of project budgets	1	Preserving a general operations budget estimation system
		2	Draw up an operations revenue and expenditure plan
		3	Project finance consulting
		4	Presentation of cash flow incurred in project work
		5	Guarantee maximum price of construction to client
4 Management services at the Middle stage	7. Cost engineering	1	System for cost estimation at the conceptual design stage
		2	System for cost estimation at the schematic design stage
		3	Draw up a plan for cost allocation
		4	Ascertain a detailed construction cost of each specialist trade
		5	An evaluating system of the contractor's bid
	8. Construction management (CM)	1	Contract management including separate contract representing client
		2	Management of cost and duration under cost-on contract
		3	Proposal and implementation of Value Engineering (VE)
		4	Providing general management on project based on the CM contract
		5	Standard forms of CM contracts providing diverse CM services
5 Design and management services at the Latter stage	9. Maintenance	1	A preserving design manual
		2	Design considering Life Cycle Cost
		3	Maintenance consulting
		4	Gathering maintenance data
		5	Creating a maintenance plan report
	10. Inspection and renewals	1	An explanation manual on operation at the delivery period
		2	Carry out a warranty investigation in the first or second year
		3	Confirming the maintenance situation after the second year
		4	Plan for long-term inspection of the building
		5	Proposal for renovations/ renewals

2. Design services at the middle stage (design and draft/ quality assurance of design).
3. Management services at the early stage (management of project team/ planning and control of project budgets)
4. Management services at the middle stage (cost engineering, construction management)
5. Design and management services at the latter stage (maintenance/ inspection and renewals)

The selection of the 50 areas of services were carried out in cooperation with professionals following the trends of the AIA and RIBA (AIA 1993, RIBA 1993), and by referring to the author's research (Furusaka 1998b, Miisho 1999a), and was produced so that it could be used as a guide for the firms to expand their provided areas. Also, as services at the latter stage are difficult to seperate into design and management, they have been grouped as one.

The Self-Diagnosis Sheet questions the service situation in the above main categories. Each question requires an answer of "1- always providing", "2- sometimes providing", or "3- not providing". For those that answered either 1 or 2, a specific comment regarding the situation is required for each. The data gathered from these five categories will be plotted on a radar-chart (Fig. 6).

Ascertaining the current situation and confirming effectiveness through the Self-Diagnosis Sheet.

Surveys using the Self-Diagnosis Sheet were carried out in 1998 and 1999 on members of the JIA. Returned sheets numbered 74, and the results were as follows:

- There is a tendency for the degree of services provided to be low in both design and management at the early stage.
- The degree of services provided was especially low in business plan/future facility plan, and the carrying out of CM services.
- Areas with a high degree of services provided were design services, and management services at the middle stage.
- This was particularly high in the area of the creation of a planning proposal report, with most firms providing them.
- There is a tendency for larger firms to provide to a wider service area (Fig. 6).
- In contrast, there is also a tendency for smaller firms to provide more to a smaller concentrated area.
- There was a tendency for smaller firms to place a particularly high weight on design and draft in design services in the middle stage. Also, firms specially concentrating on design services at the early stage in business plan/future facility plan, design and management services at the late stage, and in inspection and renewals were also noted.
- Each firm can recognize its strategy among the JIA members. The services and categories to be enhanced in future practice can be focused on. So the effectiveness of the Self-Diagnosis Sheet therefore can be confirmed, as these tendencies appear to conform to the above.

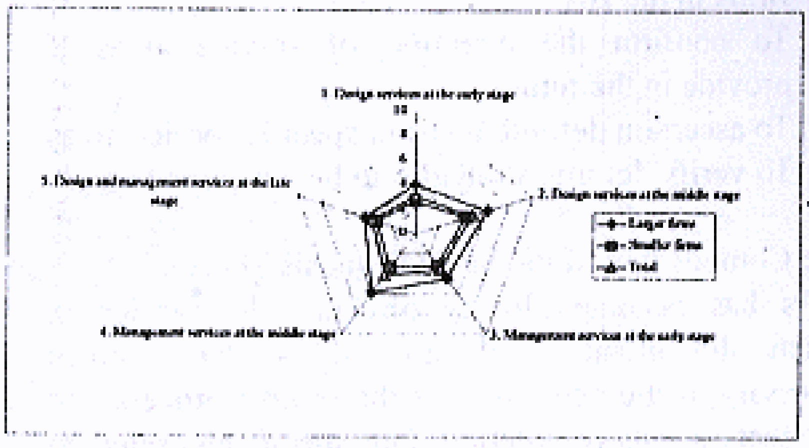

Fig. 6. Example of a Firms Self-Diagnosis

Conclusions

In this paper, the relationship between the architects' services and the clients' satisfaction was analyzed. Then, the difference of the services among each group of the architectural firms was clarified at present. Finally, the method of self diagnosis regarding the services was proposed and the behavior of the JIA members were analyzed. The findings obtained in this analysis are as follows:

- Services found unsatisfactory by the client are those deemed necessary at the beginning of the project process and after its completion, and cost-related services.
- The direction of service delivery of the JIA and the clients' satisfaction are clarified. It is possible to classify the services, which are regarded as those of "Unexpected satisfaction" and "Dissatisfaction" by the clients, into three groups to express the characteristic of each.
- A gap is found among the expectation of the client, the JIA's standard form of architects' services and the actual delivery of the services.
- The direction of service delivery by each group of architects is obtained from the analysis of their policy. The difference among their direction is found by analyzing their vision regarding the future.
- Based on the results of the structural analysis, the review of the JIA's standard is presented including the creation of new services.
- With these results, in order for the firm to create a future strategy plan a "Sheet" was created for firms to carry out a Self-Diagnosis.
- By employing this Sheet, not only could the authors ascertain trends in services provided to JIA members, but its effectiveness was also confirmed.

Acknowledgement

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Hashimoto Takayuki) established within the JIA Working Commission. The authors would like to express deepest thanks to all companies and all WG members for their cooperation in this research.

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	國立雲林科技大學 94 學年度博士班招生入學考試試題	系所：工程科技研究所 科目：工程數學（丁）

1. 求出下列微分方程式之通解： (11%)

$x^2y''-5xy'+8y=2\ln(x); \qquad (x>0)$

2. 一彈簧系統做無阻尼自由振動；彈簧係數(k)為 512 kg/m，質量(m)為 2 kg，試求其週期、頻率、及振幅。無阻尼自由振動方程式如下： (12%)

$y''+\dfrac{k}{m}y=0$ ； 初始條件為：$y(0)=0.5$ ； $y'(0)=0$

3. 試以 Laplace transform 求解下列初始值問題(initial value problem)： (12%)

$y''+2y=r(t), \quad y(0)=0, \quad y'(0)=0$

其中，$r(t)=\begin{cases}1 & if \quad 0\le t<1\\ 0 & if \quad t\ge 1\end{cases}$

4. 試求下列偏微分方程式之解？ (12%)

$\dfrac{\partial U}{\partial t}=\dfrac{\partial^2 U}{\partial x^2}+\sin(3\pi x) \qquad 0<x<1, \quad 0<t<\infty$

$B.C. \begin{matrix}U(0,t)=0\\ U(1,t)=0\end{matrix} \qquad 0<t<\infty$

$I.C. \ U(x,0)=\sin(\pi x) \quad 0<x<1$

5. 試求下列微分方程式組之解？ (12%)

$\dfrac{dx}{dt}=5x+8y+1 \qquad x(0)=4$

$\dfrac{dy}{dt}=-6x-9y+t \qquad y(0)=-3$

6. 試求下列微分方程式之解？ (11%)

$y''=(y')^3+y'$

7. 已知某廠牌電池使用壽命為常態分佈，使用壽命之標準偏差為 1.25 小時。隨機抽取該廠牌之電池 10 個，並試驗得知其樣本平均使用壽命為 40.5 小時。試問：

(1) 該樣本之檢驗結果是否足以支持該廠牌電池平均使用壽命超過 40 小時之說法(α=0.05)。上述檢定之 P 值為何。 (10%)

(2) 假如該廠牌電池真正之平均使用壽命為 42 小時，上述檢定犯第二種型式錯誤之機率為何。 (10%)

8. 已知某棉紗線之抗拉強度為常態分佈，該抗拉強度之標準偏差為 2 psi。隨機抽取該棉紗線 9 條，並試驗得知其樣本平均抗拉強度為 98 psi。試問該棉紗線真正抗拉強度之 95 %信賴區間(two-sided confidence interval)為何。 (10%)

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$

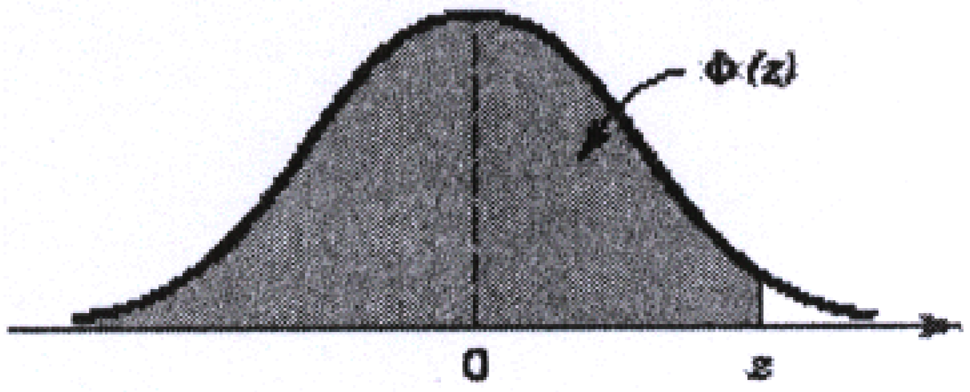

Table II Cumulative Standard Normal Distribution (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967