



1. (10%) Solve  $(y^2 - y)dx + xdy = 0$  with  $y(1) = 2$

2. (15%) Find the integrating factor and solve :

$$2 \sin y dx + \cos y dy = 0, y(0) = \frac{\pi}{2}$$

3. (10%) Solve the general solution  $y'' + 5y' + 6y = e^{-2x}$

4. (15%) Find the eigenvalues and eigenvector of the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & \sqrt{3} \\ 0 & \sqrt{3} & 6 \end{bmatrix}$$

5. (15%) Evaluate the integral

$$\oint_S [(y+z)dydz + (z+x)dzdx + (x+z)dx dy], \text{ where } S: x^2 + y^2 + z^2 = 1.$$

6. (15%) If  $f(x) = 1 - \frac{x}{2}$ ,  $0 \leq x \leq 2$ , (a) find the Fourier coefficients (with full-range expansions). (b) Find the Fourier cosine series (with half-range expansions).

7. (10%) Find the integral :  $\int \frac{\cos x \omega}{1 + \omega^2} d\omega$  .

8. (10%) Let  $\vec{F}$  be a continuous vector field with continuous first and second partial derivatives. Prove that  $\nabla \cdot (\nabla \times \vec{F}) = 0$  .