



## 1. 解常微分方程

(1)  $\frac{dy}{dx} + 3y = 3$ , when  $x = \frac{1}{3}, y = 2$  (10%)

(2)  $y'' + 5y' + 7y = \cos 3x$  (15%)

2. (1) 求  $te^{-3t} \sin 2\omega t$  之拉氏轉換 (10%)

(2) 利用拉氏轉換解  $y'' + 3y' + 5y = f(t)$

其中  $f(t) = u(t-1) + 3u(t-3), y(0) = y'(0) = 0$  (15%)

## 3. 試求所給函數的有限傅立葉正弦轉換。

(1)  $\sin(\pi - x)$  (12%)

(2)  $\sinh(\pi - x)$  (13%)

## 4. 試求下列矩陣之特徵值及特徵向量，並指出這些特徵向量是否正交。

(1)  $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$  (12%)

(2)  $\begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$  (13%)

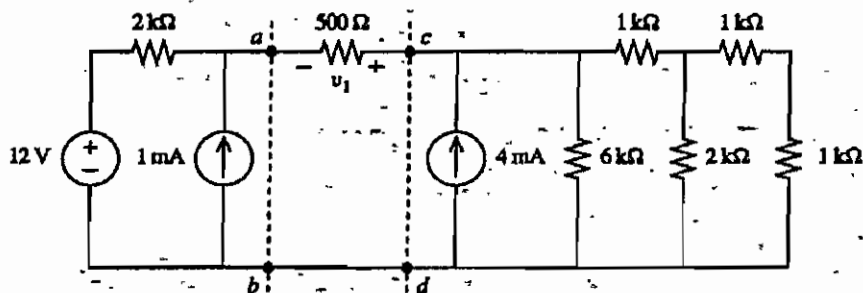


1. 參考圖一之電路。

(a) 求出圖中  $c, d$  端點右方的戴維寧等效電路。(5%)

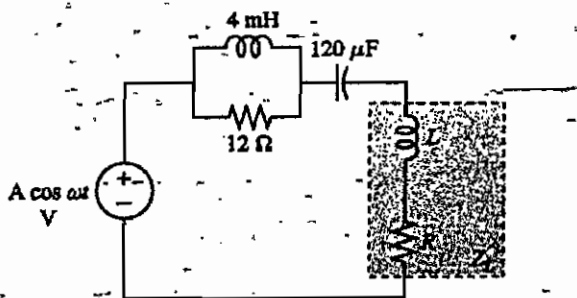
(b) 求出圖中  $a, b$  端點左方的戴維寧等效電路。(5%)

(c) 求出  $v_1$ 。(5%)



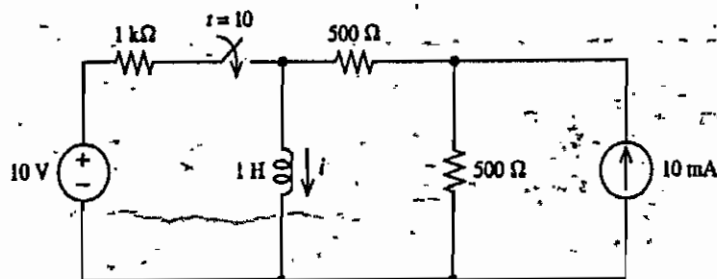
圖一

2. 圖二中，求電阻  $R$  及電感  $L$  之值，使得電源傳送最大有效功率至負載  $Z_L$ ，並計算該最大有效功率為若干。電源電壓之峰值  $A=100\sqrt{2}$ ；頻率  $\omega=1000$  rad/s。(15%)



圖二

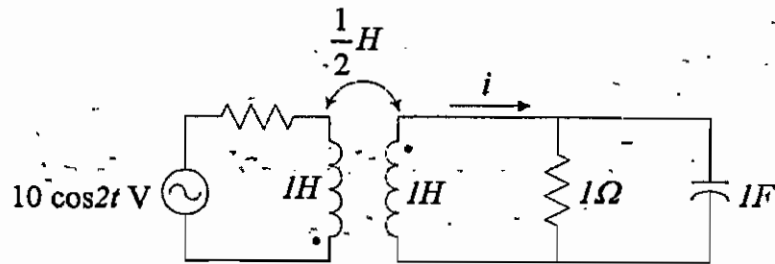
3. 參考圖三所示之電路，使用重疊定理求時間  $t > 10$  s 時之電流  $i$ ，假設在時間  $t=10$  s 時該電路已經處於直流穩態。(15%)



圖三



4. 求圖四電路之穩態電流  $i$ 。(15%)

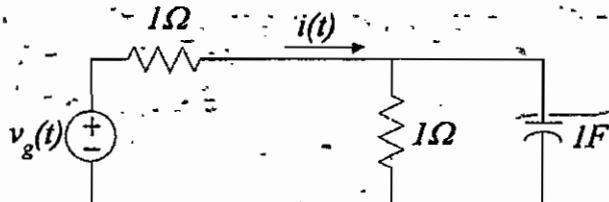


圖四

5. 圖五中之電源  $v_g(t)$  為一週期性電壓波：

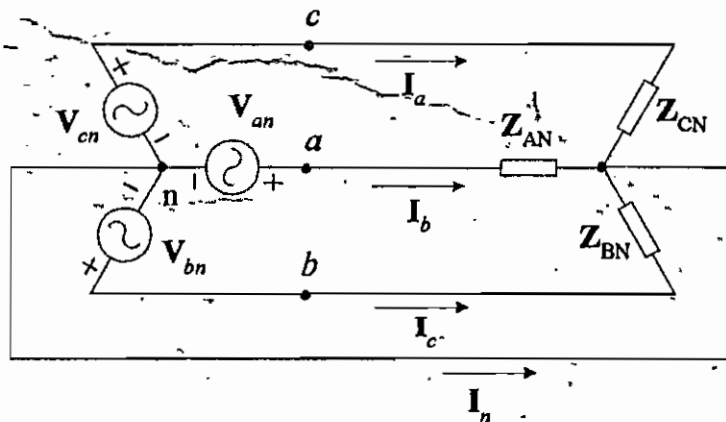
$$v_g(t) = \begin{cases} 1\text{V}, & -1 < t < 1 \\ 0\text{V}, & 1 < |t| < 2 \end{cases}, \quad v_g(t+4) = v_g(t)$$

(1) 求  $v_g(t)$  之傅立葉三角級數。(10%) (2) 求激勵響應  $i(t)$  之傅立葉三角級數的前三項。(10%)



圖五

6. 參考圖六之電路，一個 Y 接三相平衡正相序電源，以四條理想導線供應一個 Y 接的三相不平衡負載，圖中  $\bar{V}_{an} = 240/0^\circ \text{ V (rms)}$ ,  $Z_{AN} = 10 \Omega$ ,  $Z_{BN} = 10 - j5 \Omega$ ,  $Z_{CN} = j20 \Omega$ 。(1) 求中性線電流相量  $I_n$  (8%)；(2) 若中性線開路時，求三相線電流相量  $I_a$ 、 $I_b$  及  $I_c$ 。(12%) 所有電流相量請以極座標形式 (polar form) 表示。



圖六



1. Find the general solution of the following differential equation. (10%)

$$\frac{dy}{dx} = y^2 - 4xy + 4x^2 + 2$$

2. Find the Laplace transform for the following function. (10%)

$$f(t) = \cos(t-2)H(t-2) - 2tH(t-4)$$

where  $H(t)$  is Heaviside function.

3. Find the inverse Laplace transform. (10%)

$$F(s) = \frac{2se^{-2s}}{(s^2+4)^2}$$

4. A 400 gallons tank initially is half full of a fluid in which there is 50 gm of salt are dissolved. A mixture consisting of 2 gm of salt per gallon is flowing into the tank at a rate of 10 gallons per minute. Meanwhile, the brine is being drawn off simultaneously at the rate of 4 gallons per minute. Will the amount of the salt in the tank reach 600 gm before the tank is filled? (20%)

5. In  $\mathbb{R}^4$ , let  $W$  be the subset of all vectors

$$\mathbf{v} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

that satisfy  $a_4 - a_3 = a_2 - a_1$ .

- (a) Show that  $W$  is a subspace of  $\mathbb{R}^4$ . (5%)  
 (b) Show that

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

spans  $W$ . (7%)

- (c) Find a subset of  $S$  that is a basis for  $W$ . (8%)



6. Let  $L: P_1 \rightarrow P_1$  be the linear transformation defined by

$$L(at+b) = \frac{a+b}{2}t.$$

Let  $S = \{2-t, 3+t\}$  be a basis for  $P_1$ .

- Find a matrix  $A$  representing  $L$  with respect to  $S$ . (5%)
- Find the eigenvalues and eigenvectors of  $A$ . (5%)
- Find the eigenvalues and eigenvectors of  $L$ . (5%)
- Describe the eigenspace for each eigenvalue of  $L$ . (5%)

7. Consider a pulse-modulated signal modeled by the function

$$g(t) = \begin{cases} k \cos(\omega_0 t), & -a \leq t < a, \\ 0, & \text{otherwise.} \end{cases}$$

Compute the Fourier transform  $F\{g(t)\}$ . (10%)



1. Give the definition or interpretation of the following terminologies (20%)
  - (a) Diffusion current (5%)
  - (b) Series-shunt feedback amplifier (5%)
  - (c) enhancement type MOSFET (5%)
  - (d) Butterworth filter (5%)
2. For the devices in the circuit of Fig. 1,  $|V_t| = 1\text{V}$ ,  $\lambda = 0$ ,  $\gamma = 0$ ,  $\mu_n C_{ox} = 20\mu\text{A}/\text{V}^2$ ,  $L = 1\mu\text{m}$ , and  $W = 20\mu\text{m}$ . Find the values of  $I_1$  and  $V_1$ . (20%)
3. For the circuit in Fig. 2, find  $I_{O1}$  and  $I_{O2}$  in terms of  $I_{REF}$ . Assume all transistors to be matched with current gain  $\beta$ . (15%)
4. For the circuit in Fig. 3, assume that  $R = 100\text{k}\Omega$ ,  $g_m = 4\text{mA}/\text{V}$ ,  $R_L = 5\text{k}\Omega$ ,  $R_s = 100\Omega$ , and  $C_{gs} = C_{gd} = 1\text{pF}$ . Find the low-frequency gain and the upper 3-dB frequency  $\omega_H$ . (15%)
5. Find values for the resistances in the circuit of Fig. 4 such that serves as a difference amplifier with an input resistance of  $20\text{k}\Omega$  and gain of 100. (15%)
6. The 6.8 -V Zener diode in the circuit of Fig. 5 is specified to have  $V_Z = 6.8\text{V}$  at  $I_Z = 5\text{mA}$ ,  $r_Z = 20\Omega$ , and  $I_{ZK} = 0.2\text{mA}$ . The supply voltage  $V^+$  is nominally 10 V but can vary by  $\pm 1$  V. (15%)
  - (a) Find  $V_o$  with no load and with  $V^+$  at its nominal value. (5%)
  - (b) Find the change in  $V_o$  resulting from the  $\pm 1$  V. change in  $V^+$ . (5%)
  - (c) What is the minimum value of  $R_L$  for which the diode still operates in the break-down region? (5%)

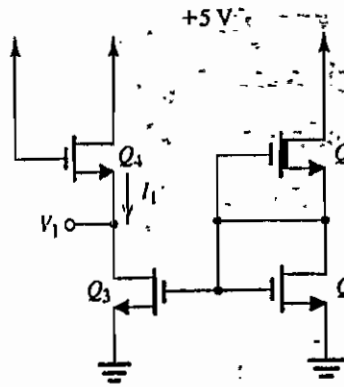


Figure 1

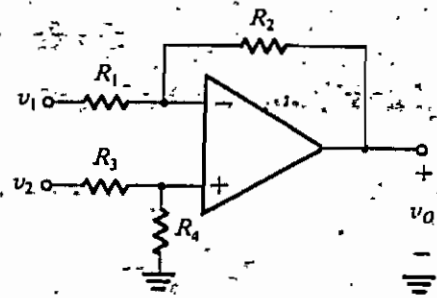


Figure 4

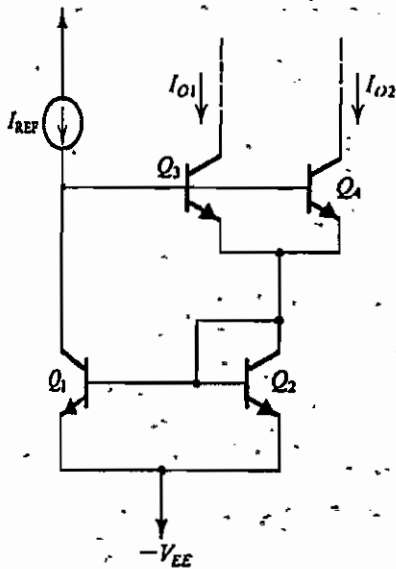


Figure 2

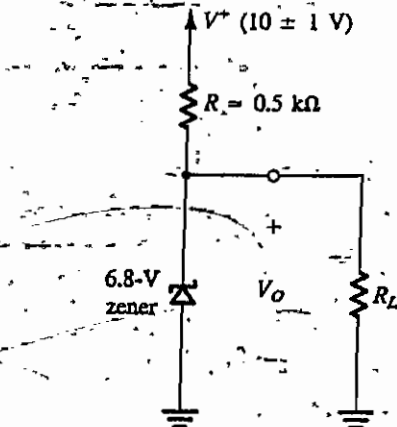


Figure 5

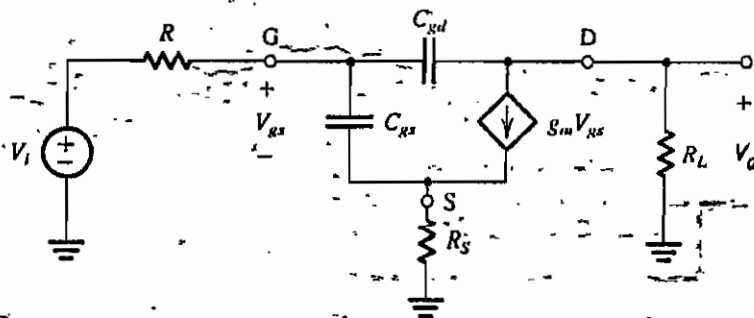


Figure 3



(1) An LC ladder network is shown in Figure 1.

(a) Construct a signal-flow graph for this network. (10%)

(b) Determine the transfer function  $\frac{V_2(s)}{V_1(s)}$  from the signal-flow graph in (a). (10%)

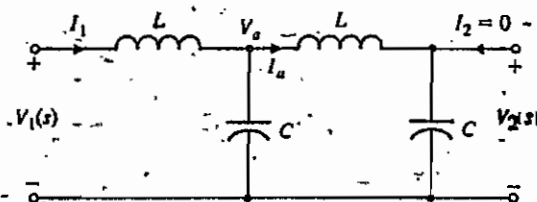


Figure 1

(2) For the network shown in Figure 2, determine the state differential equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad \text{if } \mathbf{x} = \begin{bmatrix} v_C \\ i_L \end{bmatrix} \quad \text{and } \mathbf{u} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}. \quad (15\%)$$

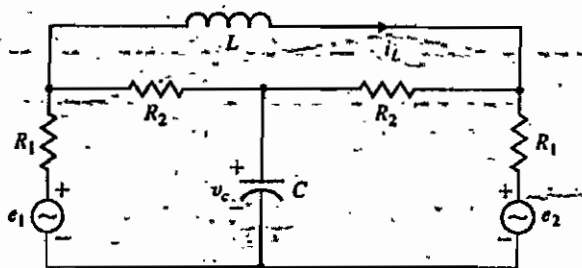


Figure 2

(3) A system is described by its characteristic equation  $s^4 + s^3 + s^2 + s + K = 0$ . Determine the range of  $K$  for the system to be stable. (15%)

(4) Consider the unity-feedback system shown in Figure 3.

(a) Find the root locus for  $\alpha > 0$  (10%)

(b) Find the  $\alpha$  such that the system has the smallest settling time and overshoot. (5%)

(c) From (b), is it a phase-lead or phase-lag network? (5%)

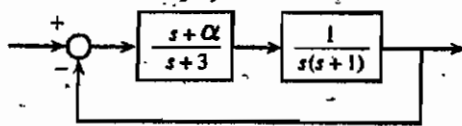


Figure 3





- (5) Consider  $G(s) = \frac{k(1+0.5s)}{s(bs+1)(s+10)}$ . Its Bode plot is plotted in Figure 4. What are  $k$  and  $b$ ? (10%)

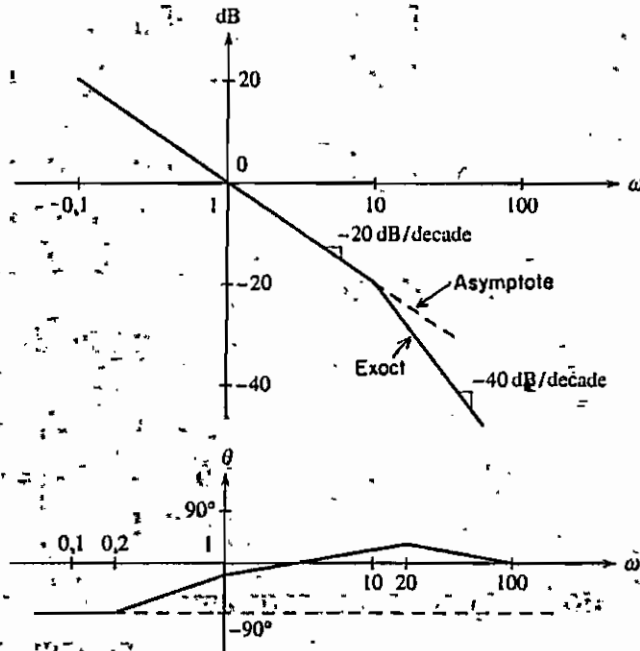


Figure 4

- (6) Consider the sampled-data system of Figure 5 for  $T = 0.1$  second.
- (a) Find the unit-step response  $c(nT)$ ? (10%)
- (b) Find the steady-state error ( $e_{ss}$ ) for the unit-ramp input. (10%)

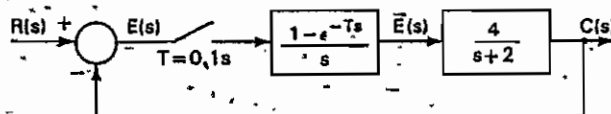


Figure 5



1. (10%) Find a least-squares solution of  $Ax=b$  for

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

2. (10%) Compute  $A^7$  where

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

3. (10%) Find the best least squares approximation to  $e^x$  on the interval  $[0,1]$  by a linear function.

4. (a) Let  $A$  be a 2-by-2 matrix and let  $p(\lambda) = \lambda^2 + b\lambda + c$  be the characteristic polynomial of  $A$ . Show that  $b = -\text{tr}(A)$  and  $c = \det(A)$ . (5%)  
 (b) Show that the matrix

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

will have complex eigenvalues if  $\theta$  is not a multiple of  $\pi$ . Give a geometric interpretation of this result. (5%)

5. (10%) For the given matrix  $A$  below, find the rank  $A$ ,  $\dim(\text{Nul } A)$ , and the bases for  $\text{Col } A$ ,  $\text{Row } A$ , and  $\text{Nul } A$ .

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$$



6.(15%) Consider a discrete random variable  $X$  whose range is the set of nonnegative integers. Let the probability distribution of  $X$  be of the form

$$p_i = P[X = i] = kp^i, \quad i = 0, 1, 2, \dots$$

where  $p$  is a given parameter,  $0 < p < 1$ .

- (a) Determine the constant  $k$ . [7%]  
 (b) Obtain the distribution function of  $X$ . [8%]

7.(15%) An error in a system can be made in three disjoint events designated  $A$ ,  $B$ , and  $C$ . Let  $E$  designate the error event, and let

$$P(E|A) = 0.03, \quad P(E|B) = 0.04, \quad P(E|C) = 0.06$$

Also,

$$P(A) = 0.3, \quad P(B) = 0.4, \quad P(C) = 0.3$$

It is given that an error occurred. What is the probability that the error is associated with  $C$ ?

8.(20%) Let us now consider the case in which a Poisson stream branches out into  $m$  output paths, as shown in Fig. 1. If the input rate is  $\lambda$  and the output path of each arrival is chosen independently with the probability  $r_k$ ,  $k = 1, 2, \dots, m$ . Show that the  $k$ th output stream is a Poisson process with rate  $r_k \lambda$ . Furthermore, these  $m$  streams are statistically independent.

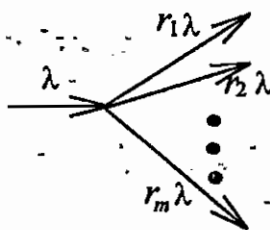


Fig. 1

(Hint: Try to use the multinomial joint distribution of the number of arrivals destined for individual  $m$  output paths in a fixed time interval.)



1. (15%) Let  $G(f)$  be the Fourier transform of  $g(t)$ . Define  $x(t) = g(2t + 1)$ . Express the Fourier transform of  $x(t)$  in terms of  $G(f)$ .
2. (10%) The signal  $s(t) = [1 + m \cos(2\pi f_m t)] \cos(2\pi f_c t)$  is detected using an ideal envelope detector. Sketch the detector output when
  - (a)  $m = 2$
  - (b)  $m = 0.5$
3. (20%) An FM radio is tuned to receive an FM broadcasting station of frequency 96.9 MHz. The radio is of the superheterodyne type with the LO operating on the high side of the input signal and using a 10.7-MHz IF amplifier.
  - (a) (6%) Determine the LO frequency.
  - (b) (8%) If the FM signal has a bandwidth of 160 kHz, determine the filter characteristics of the RF and IF filters.
  - (c) (6%) Calculate the image frequency.
4. (20%) A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to  $50 \times 10^6$  bits/s.
  - (a) What is the maximum message bandwidth for which the system operates satisfactorily?
  - (b) Determine the output signal-to-quantization noise ratio when a full-load sinusoidal modulating wave of frequency 1 MHz is applied to the input.
5. (20%) Consider a signal defined as

$$s(t) = \begin{cases} -A/2, & 0 < t < T/2 \\ A/2, & T/2 < t < T \\ 0, & \text{elsewhere} \end{cases}$$

where  $A$  and  $T$  are positive constants.

- (a) (7%) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.
  - (b) (8%) Plot the matched filter output as a function of time.
  - (c) (5%) What is the peak value of the output?
6. (15%) A delta modulator has the message signal  $m(t) = 3 \sin(200\pi t) - 2 \sin(400\pi t)$ . Determine the minimum sampling rate required to prevent slope overload, assuming that the step size of the modulator is  $\Delta = 0.2\pi$ .



1. Explain the difference between structure and behavior in the digital system context. Illustrate your answer by given
  - (a) a purely structural description
  - (b) a purely behavioral description of a full adder that computes the numerical sum of its three bits using binary arithmetic:
 
$$c_0x_0 = x_0 \text{ plus } y_0 \text{ plus } c_{-1} \quad (20\%)$$
  
2. A floating-point pipeline has four-stage  $S_1, S_2, S_3, S_4$ , whose delays are 100, 90, 85, and 80 ns, respectively.
  - (a) List the advantages and disadvantages of designing a floating-point processor in the form of a k-stage pipeline.
  - (b) What is this pipeline's maximum throughput in millions of floating-point operations per second (MFLOPS)? (20%)
  
3. The Motorola 680X0 instruction JSR SUB pushes the contents of the program counter PC onto a stack using stack pointer SP and then causing a jump to the instruction at memory location SUB. The last instruction of a subroutine should be return from subroutine (RTS) which restore to PC the address saved earlier by JSR; this instruction should also update SP. Assuming SP and PC can be used as operands of MOVE:
  - (a) Show how to use the 680X0 MOVE instructions to simulate JSR.
  - (b) Use 680X0 MOVE instruction to simulate RTS. (10%)
  
4. Explain the following terms briefly.
  - (a) Associative Memory (3%)
  - (b) Virtual Memory (3%)
  - (c) Microprogramming (3%)
  - (d) Nanoprogramming (3%)
  - (e) Programmed IO (3%)
  - (f) DMA Controllers (3%)



5. A certain processor has a microinstruction format containing 8 separate control fields  $C_0:C_7$ . Each  $C_i$  can activate any one of  $n_i$  distinct control lines, where  $n_i$  is specified as follows:  $n_0 = 2$ ,  $n_1 = 3$ ,  $n_2 = 4$ ,  $n_3 = 11$ ,  $n_4 = 9$ ,  $n_5 = 16$ ,  $n_6 = 7$ ,  $n_7 = 22$
- What is the minimum number of control bits needed to represent the 8 control fields? (4%)
  - What is the maximum number of control bits needed if a purely horizontal format is used for all the control information? (4%)
6. Suppose that the page-address trace 1, 6, 4, 5, 1, 5, 1, 4, 3, 2, 1, 2, 1, 4, 6, 7, 4, is generated by a two-level cache-main-memory scheme that uses demand paging and has a cache capacity of four pages. Assume a "hot" start, in which the cache initially has pages 1, 2, 3, and 4 allocated to it. Show the actions of the following page-replacement policies in such a page-address trace.
- FIFO (First-In-First-Out) (6%)
  - LRU (Least Recently Used) (6%)
7. Consider a two-level memory ( $M_1, M_2$ ) which has the access times  $t_{A1} = 10^{-8}$ s and  $t_{A2} = 10^{-3}$ s. Calculate the value of the hit ratio  $H$  in order for the access efficiency to be at least 60 percent of its maximum possible value. (6%)
8. The single shared bus is widely used as an interconnection medium in both sequential and parallel computers.
- What are its main advantages? (3%)
  - What are its main disadvantages? (3%)