



1. Solve the initial value problem:
 - (a) $y'' - 6y' + 9y = 0$, $y(0) = 1, y'(0) = 2$. (10%)
 - (b) $y'' - y' - 2y = e^{-x}$, $y(0) = 2, y'(0) = 2/3$. (15%)
2. Using Laplace transforms to solve the following problem:
 $\ddot{y} - y = t$, $y(0) = 1, y'(0) = 1$. (15%)
3. Find the following convolution:
 $u(t - \pi) * \cos(t)$ where $u(t) = 1$ if $t \geq 0$. (10%)
4. Find the Fourier transforms of the following function:

$$f(x) = \begin{cases} e^x & \text{if } x < 0 \\ e^{-x} & \text{if } x > 0 \end{cases} \quad (15\%)$$

5. Find the Fourier series of the following function:

$$f(x) = x^2, \quad (-\pi < x < \pi). \quad (15\%)$$

6. (a) Find the eigenvalues and the corresponding eigenvectors of the matrix $\begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$
 (10%)

(b) The eigenvectors are orthogonal? Find a matrix that diagonalizes the matrix.

(10%)



- (1) Find the general solution for the following differential equation. (10%)

$$(6x^2 - 3xy) \frac{dy}{dx} + 9xy - 2y^2 = 0$$

- (2) Find the general solution for the following differential equation. (15%)

$$y \left(\frac{d^2 y}{dx^2} \right) + 2 \left(\frac{dy}{dx} \right) = \left(\frac{dy}{dx} \right)^2$$

- (3) Use the Laplace transform to solve the following differential equation. (15%)

$$\frac{d^2 y}{dt^2} + 2t \left(\frac{dy}{dt} \right) - 4y = 6; \quad y(0) = \frac{dy}{dt}(0) = 0$$

- (4) Find the inverse Laplace transform for the function. $F(s) = \ln\left(\frac{s^2+1}{s^2+s}\right)$. (10%)

- (5) Give the definition or concept for the following terms.

(a) Linear dependence for the functions, $f_1(t), f_2(t), \dots, f_n(t)$, where $f_i(t) \in \mathbf{R}^n$. (5%)

(b) Diagonalizable matrix. (5%)

- (6) Find the Fourier transform for the following function. (10%)

$$f(t) = te^{-t}H(t), \quad \text{where } H(t) \text{ is Heaviside function.}$$

- (7) Find the inverse Fourier transform (10%)

$$F(w) = e^{-|w+4|} \cos(2w+8)$$

- (8) Show that the eigenvalues of a unitary matrix have absolute value 1. (10%)

- (9) Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, find e^A . (10%)



- (8%) Find the equation $y = a + bx$ of the least-squares line that best fits the data points $(0,1), (1,1), (2,2), (3,2)$.
- (8%) Prove that if v_1, v_2, \dots, v_r are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_r$ of an $n \times n$ matrix A then the set $\{v_1, v_2, \dots, v_r\}$ is linearly independent.
- (a)(8%) Prove the *parallelogram law* for vectors u and v in R^n :

$$\|u + v\|^2 + \|u - v\|^2 = 2\|u\|^2 + 2\|v\|^2. \text{ Do not use the Pythagorean theorem.}$$

(Hint: use inner product)

- (b)(10%) Use Pythagorean theorem to prove the *Best Approximation Theorem* which is stated as follow: Let W be a subspace of R^n , y be any vector in R^n , and y' be the orthogonal projection of y on to W determined by an orthogonal basis of W . Then y' is the closest point in W to y , in the sense that

$$\|y - y'\| < \|y - v\|$$

for all v in W distinct from y' .

- (8%) Prove that if $S = \{u_1, \dots, u_p\}$ is an orthogonal set of nonzero vectors in R^n , then S is linearly independent and hence is a basis for the subspace spanned by S .
- (8%) The set $S = \{u_1, u_2, u_3\}$ is an orthogonal basis for R^3 , where

$$u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$$

Express the vector $y = \begin{bmatrix} 6 \\ 1 \\ -8 \end{bmatrix}$ as a linear combination of the vectors in S .



6.(15%) Suppose that two dice are thrown and that the dice are distinguishable. An outcome of this experiment is denoted by (m, n) , where m and n are the faces of the dice. Let A and B be the following events of this experiment:

$$A = \{m + n = 11\},$$

$$B = \{n \neq 5\}.$$

(a) Find the probability of occurrence of the event A , $P[A]$. [4%]

(b) Find the probability of occurrence of the event B , $P[B]$. [4%]

(c) Show that $P[A, B] \neq P[A]P[B]$, [7%]

where $P[A, B]$ is the joint probability of events A and B .

7.(15%) A coin with $P[\text{head}] = p$, $P[\text{tail}] = q$, $p + q = 1$ is tossed n times.

If $n \rightarrow \infty$, $p \rightarrow 0$, $np \rightarrow a$ (a finite value of the order of 1),

show that

$$P[k \text{ heads in } n \text{ tossings}] \cong \frac{a^k}{k!} e^{-a}.$$

8.(20%) A device is put into service at time zero, and then we follow it until it fails.

T is the random variable we use for the time of failure. We assume that the failure of electrical components is modeled by the exponential random variable with the pdf $f(t) = \lambda e^{-\lambda t}$ where λ is called a rate parameter because it has the units of inverse time.

(a) Reliability $R(t)$ is defined as the probability that a device will fail after time t .

Show that $R(t) = e^{-\lambda t}$. [7%]

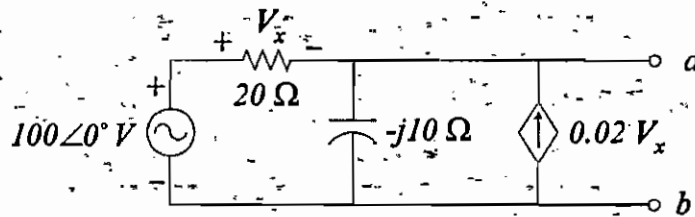
(b) Show that the mean time to failure (MTTF) is given as

$$MTTF = E[T] = 1/\lambda. \quad [7\%]$$

(c) Show that the probability that a device will provide at least MTTF of use before it fails is e^{-1} ($=0.368$). [6%]

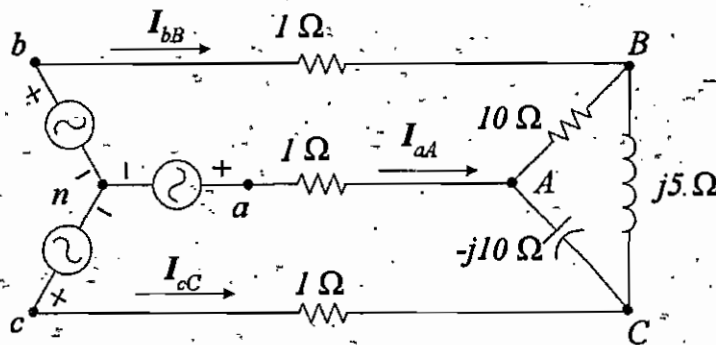


1. 戴維寧定理 參考圖一之交流電路，試求自 a, b 端點看入電路的戴維寧等效電路 V_{th} 及 Z_{in} 之值，請以極座標形式 (polar form) 表示。(20%)。



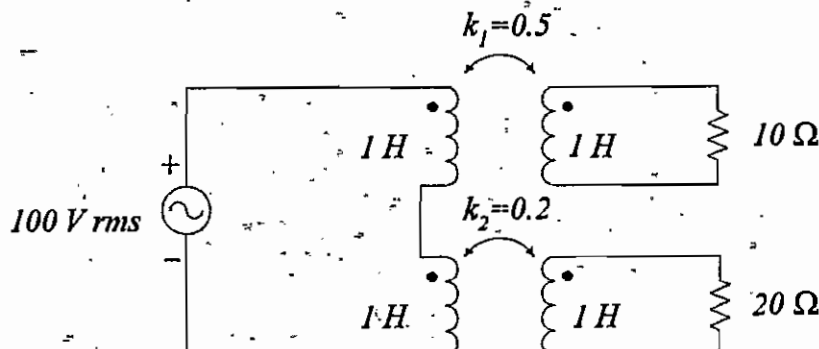
圖一

2. 三相交流分析 參考圖二之三相電路，平衡 Y 接三相電源 $V_{an} = 120\angle 0^\circ$ V rms, $V_{bn} = 120\angle -120^\circ$ V rms, $V_{cn} = 120\angle 120^\circ$ V rms。求三相線電流 I_{aA} 、 I_{bB} 及 I_{cC} ，請用極座標形式表示 (30%)。



圖二

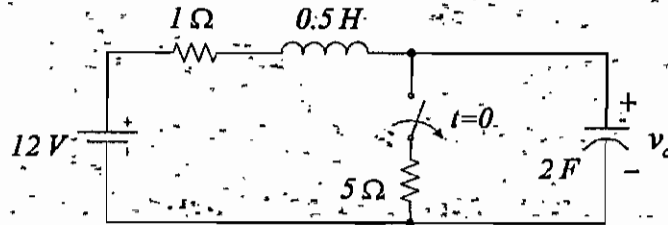
3. 磁耦合電路 參考圖三之交流電路，設角頻率 $\omega = 100$ rad/s, k_1 及 k_2 表示耦合係數，計算 (a) 10Ω 電阻所吸收之有效功率 (10%)；(b) 電源所提供的無效功率 (10%)。



圖三



4. 暫態分析 圖四所示電路的開關接通已有一段很長的時間，在 $t=0$ 時開關打開，試求 $t>0$ 時電容器兩端之電壓 $v_c(t)$ (30%)。



圖四



1. For the amplifier circuit in Fig. 1, assume that V_s has a zero dc component and the BJTs have $\beta = 100$. Find V_o/V_s and R_{in} . (25%)
2. For the circuit in Fig. 2 in which the transistors have $V_{BE} = 0.7\text{V}$ and $\beta = 100$, find i_c , v_o/v_i , and R_{in} . (25%)
3. The NMOS transistors in the circuit of Fig. 3 have $V_t = 2\text{V}$, $\mu_n C_{ox} = 20\mu\text{A/V}^2$, $\lambda = 0$; and $L_1 = L_2 = 10\mu\text{m}$. Find the required values of gate width for each of Q_1 and Q_2 , and the value of R , to obtain the voltages and current values indicated. (25%)
4. The amplifier in Fig. 4 is biased to operate at $I_D = 1\text{mA}$ and $g_m = 1\text{mA/V}$. Neglecting r_o , find the midband gain. Find the value of C_S that places the corresponding pole at 10 Hz. What is the frequency of the transfer-function zero introduced by C_S ? Give an expression for the gain function $V_o(s)/V_i(s)$. What is the gain of the amplifier at dc. (25%)



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系所：電機系

科目：電子學

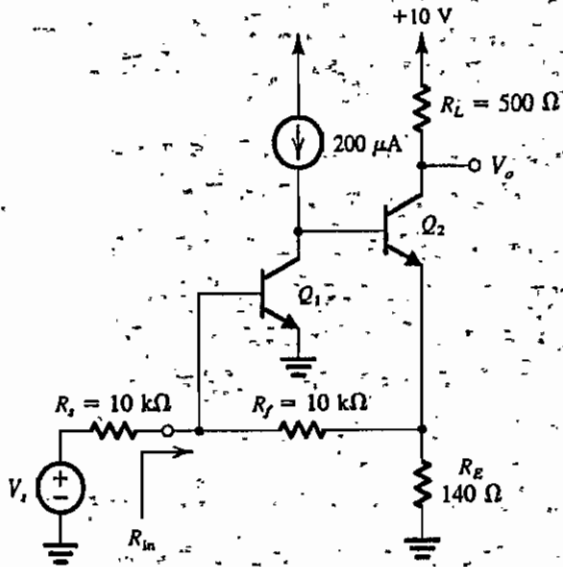


Fig. 1

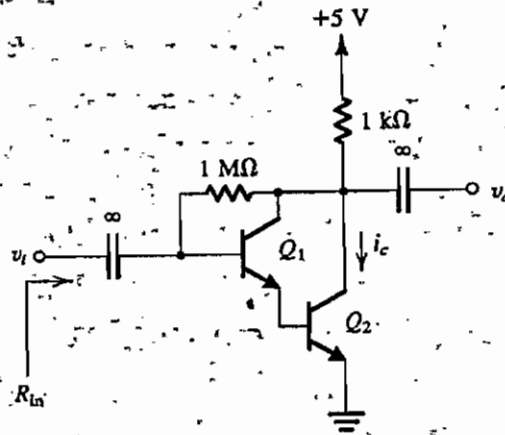


Fig. 2

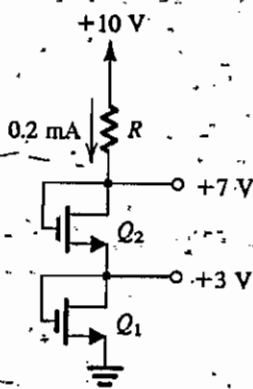


Fig. 3

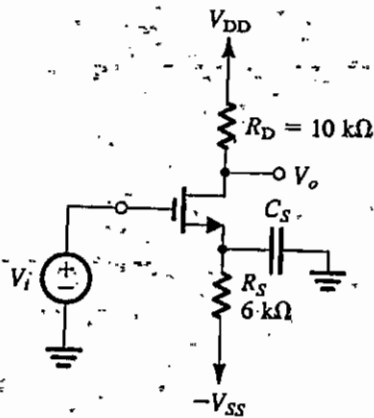


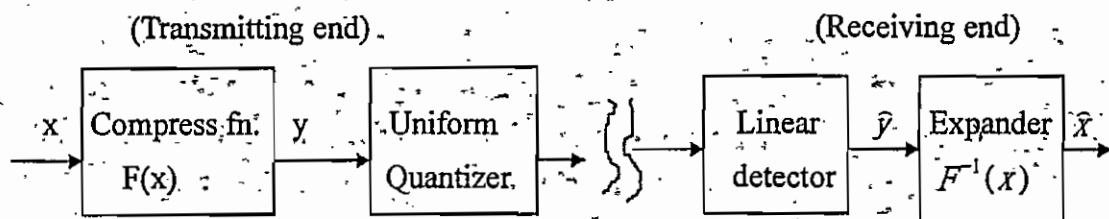
Fig. 4



- (1) (25%) A signal $x(t)$ of finite energy is applied to a system with output $y(t) = x^2(t)$. The spectrum of the $x(t)$ is limited to the frequency interval $-W \leq f \leq +W$. Show that the spectrum of $y(t)$ is limited to $-2W \leq f \leq +2W$.
 Hint: Express $y(t)$ as $x(t)$ multiplied by itself.
- (2) (25%) Given a signal $x(t) = \cos(\omega_0 t)$ which is sampled by an impulse train $p(t)$ at $\omega_s = 2\pi/T = 600$; where T is the time interval between sampling impulses. An ideal lowpass filter $H(j\omega)$ with gain T and cut-off frequency $\omega_c = 300$ (include 300) is used in the reconstruction process. What is the reconstructed signal $x_r(t)$ at the following ω_0 ?
 (a) $\omega_0 = 200$ (b) $\omega_0 = 400$
 Explain your answer using sampling theorem and Fourier transform $X_r(j\omega)$

(3) 25%

A companding system is shown below.



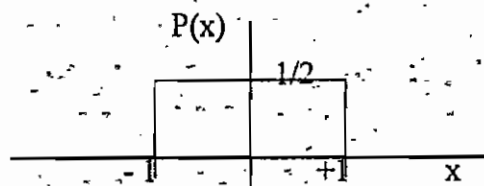
5% (A) Find the mean square error (MSE) for the uniform quantizer, assume that the step size is Δ .

10% (B) Describe why that the MSE of the above system can be approximated as

$$\bar{\epsilon}_x \approx \frac{\Delta^2}{12} \int \frac{p(x)}{[F'(x)]^2} dx$$

where $p(x)$ is the probability density function of the signal X .

10% (C) Now suppose we use 4-bit uniform quantizer and $p(x)$ is given as the following figure.

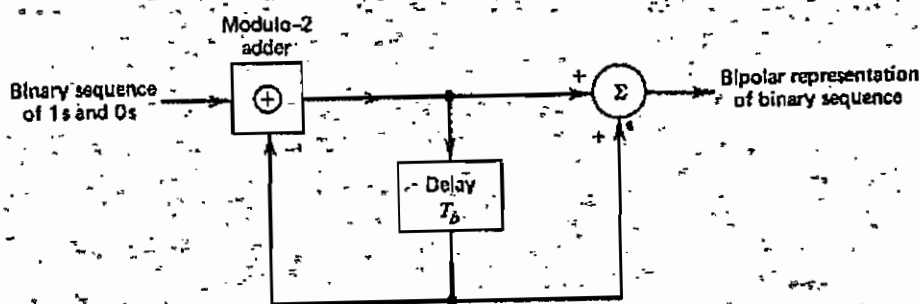


and $F(x)$ is given as $F(x) = \ln(|x|) \text{sgn}(x)$. Find the SNR.



(4) 15%

The scheme shown in the following figure may be viewed as a differential encoder (consisting of the modulo-2 adder and the 1-unit delay element) connected in cascade with a special form of correlative coder (consisting of the 1-unit delay element and summer). A single delay element is shown in this figure since it is common to both the differential encoder and the correlative coder. In this differential encoder, a transition is represented by symbol 0 and no transition by symbol 1.



5% (A) Find the frequency response and the impulse response of the correlative coder

part of the scheme shown in the figure.

5% (B) Show that this scheme may be used to convert the on-off representation of a binary sequence (applied to the input) into the bipolar representation of the sequence at the output. You may illustrate this conversion by considering the sequence 010001101.

5% (C) Consider a random binary wave $x(t)$ in which the 1s and 0s occur with equal probability, the symbols in adjacent time slots are statistically independent, and symbol 1 is represented by A volts and symbol 0 by zero volts. This on-off binary wave is applied to the circuit in the figure. Show the power spectral density of the bipolar wave $y(t)$ appearing at the output of the circuit equals

$$S_Y(f) = T_b A^2 \sin^2(\pi f T_b) \operatorname{sinc}^2(\pi f T_b)$$

(5) 10%

5% (A) Please describe the advantages of using the GMSK modulation scheme.

5% (B) Draw a simple GMSK modulator and demodulator.



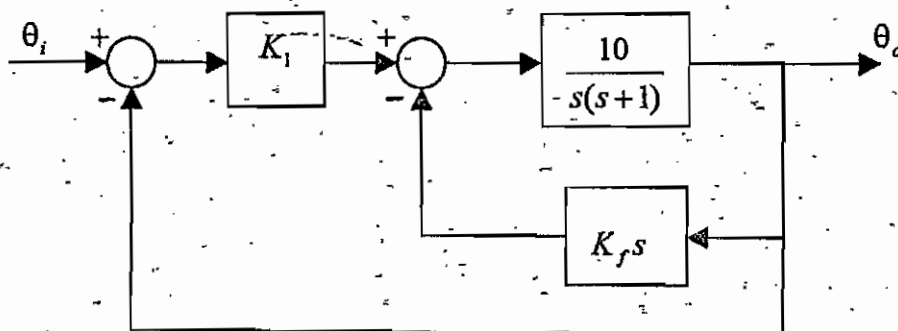
01. Explain the following terms briefly.
- (a) Hypercube. (3%)
 - (b) Fault Tolerance (3%)
 - (c) RAID (3%)
02. Suppose that a certain magnetic hard disk drive has the following specifications:
- | | |
|---|---------------|
| Number of disks (recording surfaces) | 14(27) |
| Number of tracks per recording surface | 4925 |
| Number of sectors on all recording surfaces | 17,755,614 |
| Storage capacity (formatted) of disk drive | 9.09 GB |
| Disk-rotation speed | 5400 rev/min |
| Average seek time | 11.5 ms |
| Internal data-transfer rate | 44 to 65 MB/s |
- (a) What is the block size? (4%)
 - (b) What is the average rotational latency? (4%)
 - (c) What is the average block access time? (4%)
03. Design a counter with the following repeated binary sequence: 0, 1, 3, 7, 6, 4. Use T flip-flops. Treat the unused states as don't-care conditions. Analyze the final circuit to ensure that it is self-correcting. If your design produces a nonself-correcting counter, you must modify the circuit to make it self-correcting. (12%)
04. (a) How many 128×8 RAM chips are required to support a memory capacity of 2048 bytes? (2%)
- (b) How many lines of the address must be used to access 2048 bytes? How many of these lines are connected to the address inputs of all chips? (4%)
- (c) How many lines must be decoded for the chip select inputs? Specify the size of the decoder. (4%)
05. Design a parallel priority interrupt hardware for a system with four interrupt sources. (7%)



06. The following binary word $W=10001011$ is stored in an 8-bit register. What is the decimal number represented by W if it is interpreted, as an integer in each of the following codes: (15%)
- (a) Unsigned binary
 - (b) Signed binary (two's complement)
 - (c) Sign-magnitude
07. Analyze the three bus-arbitration methods (daisy chaining, polling, and independent requesting) with respect to communication reliability in the event of hardware failures. (15%)
08. Define each of the following IO control methods: programmed IO, DMA controllers, IOPs (input-output processor). List the advantages and disadvantages of each method with respect to program-design complexity, IO bandwidth, and interface hardware costs. (20%)



1. 一系統之轉移函數為 $G(s) = \frac{1}{(s+1)(s+5)}$ ，求此系統對輸入 $u(t) = 10 \sin(2t + 45^\circ)$ 的穩態輸出響應。(20%)
2. 一系統的轉移函數為 $G(s) = \frac{s^2 + 6s + 8}{s^3 + 6s^2 + 11s + 6}$ ，求此系統的狀態方程式與輸出方程式，使系統分別為
- 狀態可控制 (controllable)。(10%)
 - 狀態可觀測 (observable)。(10%)
 - 狀態可控制且可觀測 (controllable and observable)；若不能達到此要求，則解釋原因為何。(10%)
3. 圖(一)所示的控制系統，試設計 K_1 與 K_f 值，使得當輸入為單位斜坡函數 (unit ramp function) $\theta_i(t) = t$ ($t \geq 0$) 時，閉迴路系統具有穩態誤差 (steady-state error) $e_{ramp}(\infty) = 0.1$ 及阻尼比 (damping ratio) $\zeta = 0.5$ 。(20%)



圖(一)

4. 考慮圖(二)的回授控制系統，其中

$$G(s) = \frac{1}{(s+2)(s^2 + 2s + 2)}$$



- (a) 應用奈奎氏準則 (Nyquist criterion) 決定系統穩定的 K 值範圍。 (10%)
- (b) 試以羅斯 - 赫維茲 (Routh-Hurwitz) 法則驗證(a)小題中的答案。 (10%)
- (c) 當 $K = 6$ 時，求系統的增益邊際 (gain margin)。 (10%)

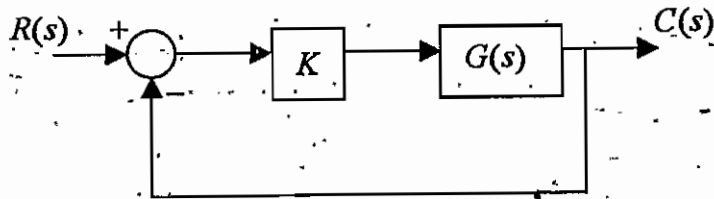


圖 (二)