



1. (20%) A 90% modulated AM wave is written as  $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$ , where the message signal is  $m(t) = \cos(2 \times 10^4 \pi t) + 0.5 \cos(3 \times 10^4 \pi t)$ . The carrier amplitude and frequency are  $A_c = 2$  and  $f_c = 1$  MHz, respectively.

- (a) (5%) Calculate the amplitude sensitivity,  $k_a$ .
- (b) (5%) Plot the spectrum of the modulated signal  $s(t)$ .
- (c) (3%) Determine the transmission bandwidth of  $s(t)$ .
- (d) (7%) Pass the modulated AM wave through an ideal low-pass filter with cut-off frequency 995 kHz. Determine the filter output (i.e., write the mathematical expression of the output).

2. (15%) A narrowband frequency modulator (NBFM) is supplied with a carrier of frequency  $f_1 = 2$  MHz and modulation index  $\beta_1 = 0.1$ . Now you are asked to use one frequency multiplier and one mixer to modify the NBFM output into a wideband (WBFM) signal with carrier frequency 96 MHz and  $\beta = 8$ . And, in order to lower down the cost, you are not allowed to use oscillators with output frequency higher than 5 MHz. Draw and specify the block diagram of your design.

3. (25%) The transmitted signals of a quaternary system (i.e.  $M = 4$ ) are defined as

$$s_i(t) = a_i \phi_1(t) + b_i \phi_2(t), \quad 0 < t < T,$$

where  $\phi_1(t)$  and  $\phi_2(t)$  are a set of orthonormal basis functions. The four possible  $(a_i, b_i)$  pairs are (0, 1), (1, 0), (1, 2), and (2, 1).

- (a) (5%) Draw the signal constellation and point out the decision region for each of the signals.
- (b) (5%) Evaluate the average transmitted energy per symbol.
- (c) (5%) Assume the signal is Gray encoded. Calculate the average probability of bit error when the signals are transmitted through an AWGN channel.
- (d) (10%) Design and plot the structure of the coherent detector.





4. (20%) The binary data stream {01 11 10 00 00} is applied to a  $\pi/4$ -shifted DQPSK modulator that is initially in the state ( $\phi_1 = \sqrt{E}$ ,  $\phi_2 = 0$ ).

- (a) (7%) By defining the relationship between input dibits and carrier-phase shifts, determine the phase states occupied by the modulator in response to the specified data stream.
- (b) (5%) Describe the advantages of a  $\pi/4$ -shifted DQPSK system over an ordinary coherent QPSK system.
- (c) (8%) Plot the block diagram of the  $\pi/4$ -shifted DQPSK detector and briefly describe how it works.

5. (20%) Figure 1 shows a four-stage shift register used for generating a spreading function in a spread spectrum (SS) system. The initial state of the register is 1000.

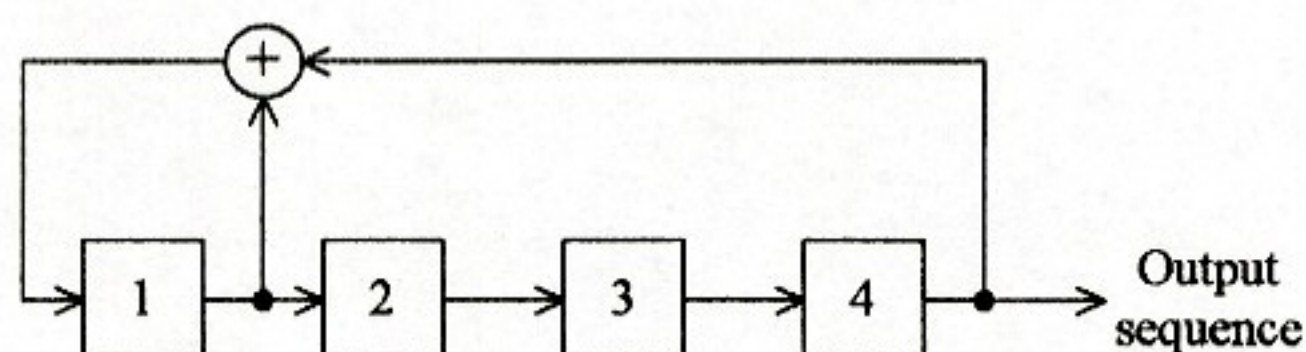


Figure 1

- (a) (5%) Find the output sequence of the shift register. Is it a maximal-length sequence?
- (b) (5%) Plot the autocorrelation function of the output.
- (c) (5%) Draw the block diagram of a DSSS/BPSK system (including the transmitter and receiver) and describe (in detail) how the system works.
- (d) (5%) It is already known that, for a coherent binary PSK receiver without SS processing,  $E_b/N_0 = 10$  yields  $P_b = 0.387 \times 10^{-5}$ . To achieve the same error rate, what is the highest tolerable interference power? Assume the received power in our DSSS system is 1 mW.





## 1. 遞迴函數

- (a) 試撰寫一 C 語言程式，以遞迴函數方式列印出 2 的  $n$  次方之值。(5%)
- (b) 撰寫上述遞迴函數時，須考慮的五個因素及其解答為何？(5%)

2. 設  $X[n][n]$  為一個下三角矩陣，

- (a) 若要用一個一維陣列  $Y[m]$  來表示  $X$ ，則  $m$  的最小值為何？(5%)
- (b) 若  $X[0][0]$  在記憶體之起始位置為  $b$ ，且每個元素所需記憶體空間為  $z$ ，請問元素  $X[i][j]$  所在之記憶體位址為何(row major)？(5%)

3. 已知稀疏矩陣(sparse matrix)  $A$ ，

$$\begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \end{matrix}$$

- (a) 請以三行式(3-tuple)表示法來表示稀疏矩陣  $A$ 。(5%)
- (b) 請以環狀鏈結串列(circularly linked list)來表示稀疏矩陣  $A$ 。(5%)
- (c) 請說明以三行式表示法及環狀鏈結串列來表示稀疏矩陣的優缺點。(5%)

## 4. 優先貯列(priority queue)

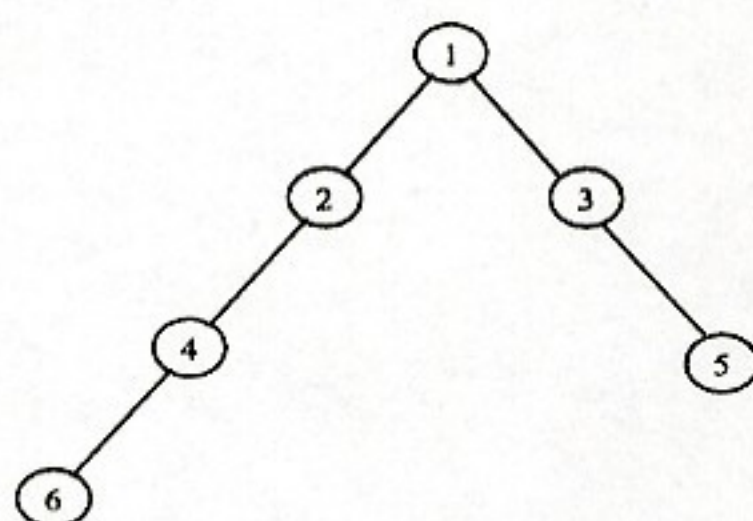
- (a) 試以鏈結串列(linked list)來實現一個具有 A B C 3 等級之優先貯列 (請自行作合理之假設並畫出資料結構圖)。(5%)
- (b) 請說明增加及刪除一元素於 B 貯列之步驟。(5%)
- (c) 請比較以陣列(array)與鏈結串列(linked list) 來實現優先貯列之優缺點。(5%)





5. 在資料結構中，

- (a) 樹(tree)與圖(graph)有何不同？(3%)
- (b) 若將圖(一)當作樹，請分別用陣列(可為一維或二維)與鏈結串列來表示它。(6%)
- (c) 若將圖(一)當作圖，請分別用陣列(可為一維或二維)與鏈結串列來表示它。(6%)



圖(一)

6. 給予如下資料序列：

15, 7, 8, 9, 2, 3

- (a) 請以氣泡排序法(bubble sort)來將它們由小到大排序。(6%)
- (b) 請分析氣泡排序法在平均情況和最差情況之時間複雜度。(6%)

7. (a) 何謂費氏樹(Fibonacci tree)? (4%)

- (b) 令費氏數列是由0開始，請畫出一棵含有20個節點的費氏樹(節點的編號為 1, 2, ..., 20)。(4%)

8. (a) 何謂最大堆積(max heap)? (4%)

- (b) 請採用由上而下(top down)的方法為下列資料序列建立一棵最大堆積。(6%)

15, 7, 8, 9, 2, 3, 6, 17, 12, 19

- (c) 當刪除19之後，此最大堆積將變成怎樣？(5%)





1. 布林函數  $f = wxy' + y'z + w'yz' + x'yz' + wxyz'$

布林函數  $g = (w + x + y' + z')(x' + y' + z)(w' + y + z')(w' + x' + y + z)$

- (3%) 先畫出布林函數  $f$  的卡諾圖(Karnaugh Map) ?
- (3%) 再畫出布林函數  $g$  的卡諾圖(Karnaugh Map) ?
- (4%) 以卡諾圖化簡方式來求出布林函數  $(f \cdot g)$  ? (  $\cdot$  是指 AND )

2. 假設一個計算機的浮點數(Floating-Point Number)的型式如圖一所示：

Bit 0	Bit 1	Bit 5	Bit 6	Bit 15
S	C	Mantissa		

圖一

其中  $S = 0$  時為正號， $S = 1$  時為負號， $C = \text{Exponent} + 16$ ，基數為 2，小數點在 Mantissa 的最左端，而且小數點右邊的第一位元恆不為 0，請問：

- (3%) 浮點數的精確度(Precision)為多少位元？
- (3%) 指數(Exponent)的範圍值為何？
- (4%) 此浮點數型式所能表達的最大正數為何？最小的正數為何？

3. 在不同的計算機中，其資料在記憶體內的儲存方式有兩種，一種是 Intel x86 CPU 所採用的 Little Endian 的方式，另外一種是 Apple 及 Sparc CPU 所採用的 Big Endian 的方式。

- (4%) 請說明 Little Endian 與 Big Endian 儲存方式的不同之處？
- (6%) 假設有一個資料的內容為  $0x12345678$ ，在 Little Endian 與 Big Endian 儲存方式的記憶體中的位元組(Byte)配置為何？請用圖二的表格來表示：

	Byte at address 3
	Byte at address 2
	Byte at address 1
	Byte at address 0

Little Endian

	Byte at address 3
	Byte at address 2
	Byte at address 1
	Byte at address 0

Big Endian

圖二

4. 請設計一個四位元比較器(Comparator)的邏輯電路(Logic Gate Circuit)，此比較器有二個輸入訊號與一個輸出訊號。二個輸入訊號皆為四位元的資料，而輸出訊號為一位元的資料。





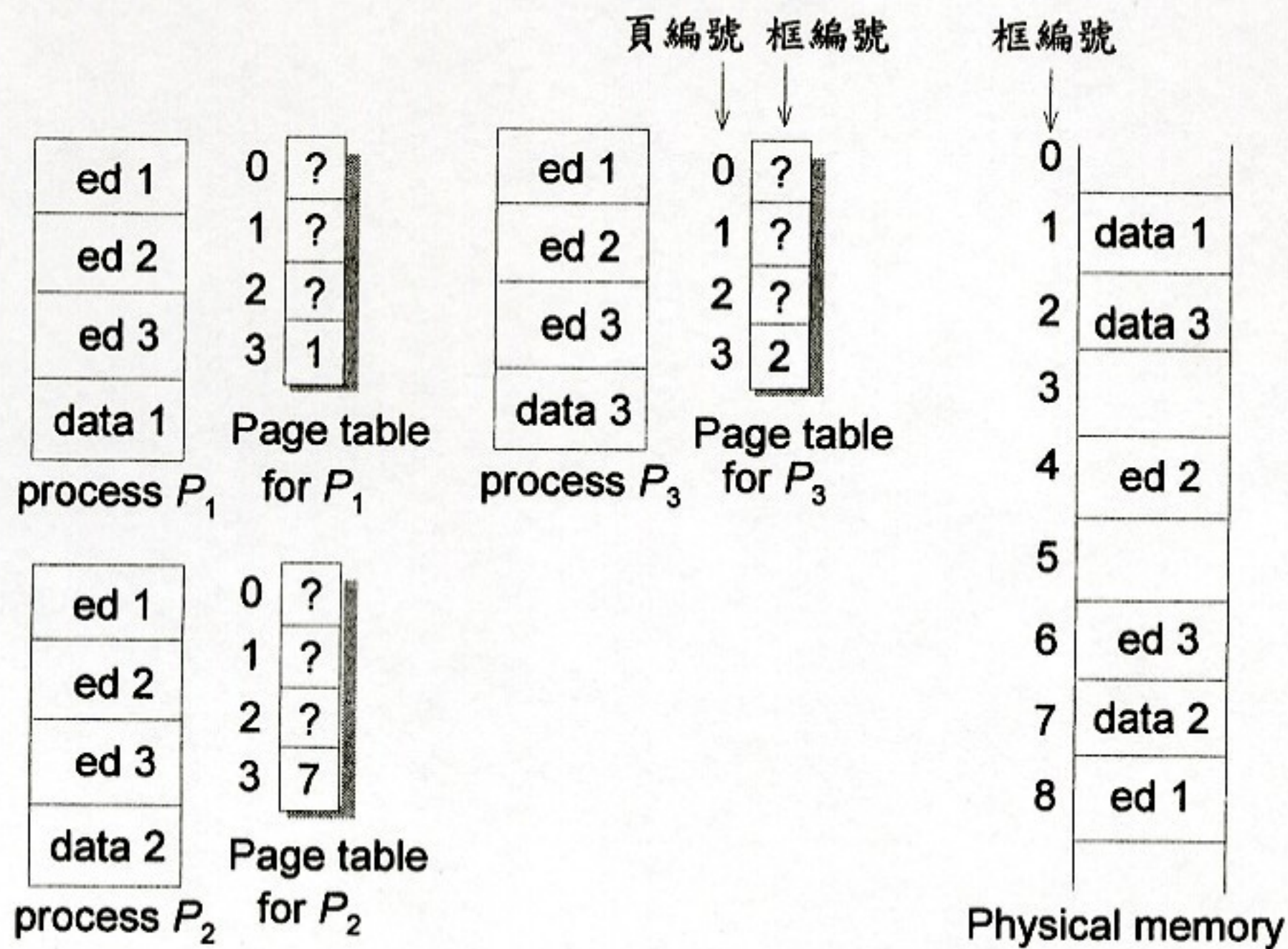
當二個輸入訊號的位元值皆相同時，比較器的輸出訊號為‘1’；當二個輸入訊號的位元值，有一個以上的位元不相同時，比較器的輸出訊號為‘0’。

- (a) (3%) 請列出比較器的輸入訊號 ( $X_i, Y_i$ ) 與輸出訊號 ( $Z = F(Z_i |_{i=0,1,2,3})$ ) 之間的真值表關係？
  - (b) (3%) 請依據此真值表，說明如何利用 XNOR 邏輯閘與 AND 邏輯閘來設計出此四位元比較器的邏輯電路？
  - (c) (4%) 由於要提高此四位元比較器的輸出準位與響應頻率，輸出端的邏輯閘電路必須改用 NOR 邏輯閘，請重新設計此四位元比較器的邏輯電路？
5. 計算機在處理數學運算式時，為了方便處理「運算子優先順序」(Operator Precedence)，並且可以利用簡單的堆疊(Stack)存取架構，通常會將傳統運算式(Infix Expression)轉換成為後置式運算式(Postfix Expression)，再來做運算處理。以  $2*3 - 4/5$  運算式為例：
- (a) (4%) 先將此傳統運算式的運算式樹(Expression Tree)畫出？
  - (b) (6%) 再依據運算式樹的規則，說明如何產生  $2*3 - 4/5$  的後置式運算式的結果？
6. (a) (5%) 處理中斷 (interrupt handling) 的步驟為何？
- (b) (5%) 試說明中斷 (interrupt) 與陷阱 (trap) 之間的差異？
7. (a) (3%) 何謂 Direct Memory Access (DMA) 的技術？
- (b) (2%) DMA 會導致 cycle stealing，原因為何？
- (c) (5%) DMA 的動作流程為何？
8. (a) (5%) 試比較編譯程式 (compiler) 與直譯程式 (interpreter) 二者之間的差異及其優缺點。
- (b) (5%) Java 環境中的 Just In Time (JIT) 編譯程式的主要功用為何？





9. (9%) 某一文書編輯器程式 (text editor) 由 150 KB 的 reentrant code 及 50 KB 的資料空間 (data space) 組成。此程式將由三個使用者程序 (user process) 同時共享使用。假設每一頁 (page) 的大小為 50 KB。則在圖三中，各個程序之



圖三

分頁表 (page table) 紀錄各頁所對應的記憶體框 (frame) 編號應分別為何?

10. 考慮圖四所示之 C 語言程式。若 `num[0][0]` 的記憶體位址為 `0x10000000`，且令每個整數型態資料均佔用 4 位元組 (bytes) 記憶體空間。

```
#include <stdio.h>
void main()
{
    int i, j, num[5][5] = {0};
    static char str[] = "YunTech EE Graduate School";
    int **ptr1 = &num[2];
    char *ptr2 = &str;

    for (i = 0; i < 5; i++)
        for (j = 0; j <= i; j++) num[i][j] = i*i+j*j;
    printf("%d, %s\n", *ptr1, ptr2+8);
}
```

圖四

- (a) (6%) 程式最末一行之 `printf()` 輸出結果為何?





國立雲林科技大學

94 學年度碩士班入學招生考試試題

系所：電機系

科目：計算機概論 (戊)

- (b) (3%) ptr1 為一指標 (pointer) 型態的變數，此變數所儲存的内容值為何？
- (c) (2%) 元素 num[2][3] 之值為何？





1. (15%) A double-circuit, three-phase, transposed transmission line is shown in **Fig. 1**. The radius of each conductor is 1.25 cm. Calculate the inductance per kilometer per phase.

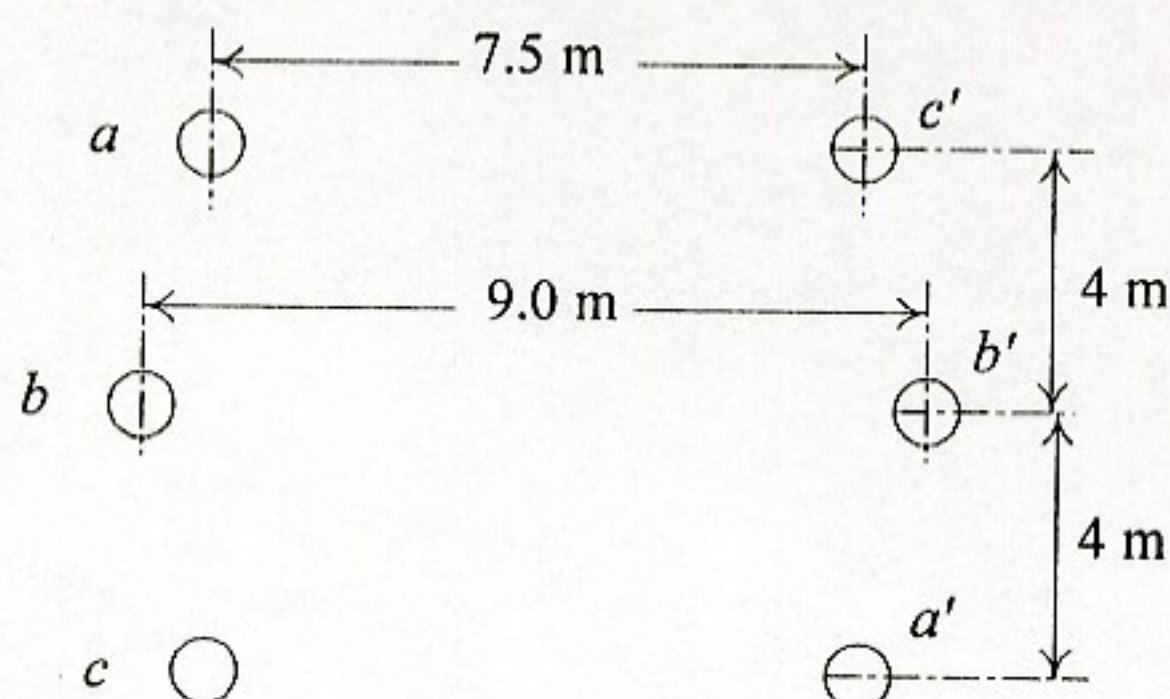


Fig. 1

2. (15%) The sending-end and receiving-end voltages of a three-phase short transmission line are  $V_s=33$  kV and  $V_R=31.2$  kV, respectively. The per-phase line parameters are  $R=10\ \Omega$  and  $X_L=20\ \Omega$ . Calculate the maximum power that can be transmitted by the line.
3. (15%) The positive-, negative-, and zero-sequence reactances of a 15-MVA, 11-kV, three-phase, Y-connected generator are 11%, 8%, and 3%, respectively. The neutral of the generator is grounded, and the generator is excited to the rated voltage on open-circuit. A line-to-ground fault occurs on phase  $a$  of the generator. Calculate the phase voltages  $V_a$ ,  $V_b$  and  $V_c$  and currents  $I_a$ ,  $I_b$  and  $I_c$ .
4. (20%) A double line-to-ground fault from phase  $b$  to  $c$  occurs at F in the system shown in **Fig. 2**. (a) Draw the sequence networks for the system (10%), and (b) calculate the line current  $I_b$  (10%).
5. (20%) Find  $Y_{bus}$  and  $Z_{bus}$  for the network shown in **Fig. 3**. All impedances are in per-unit values.



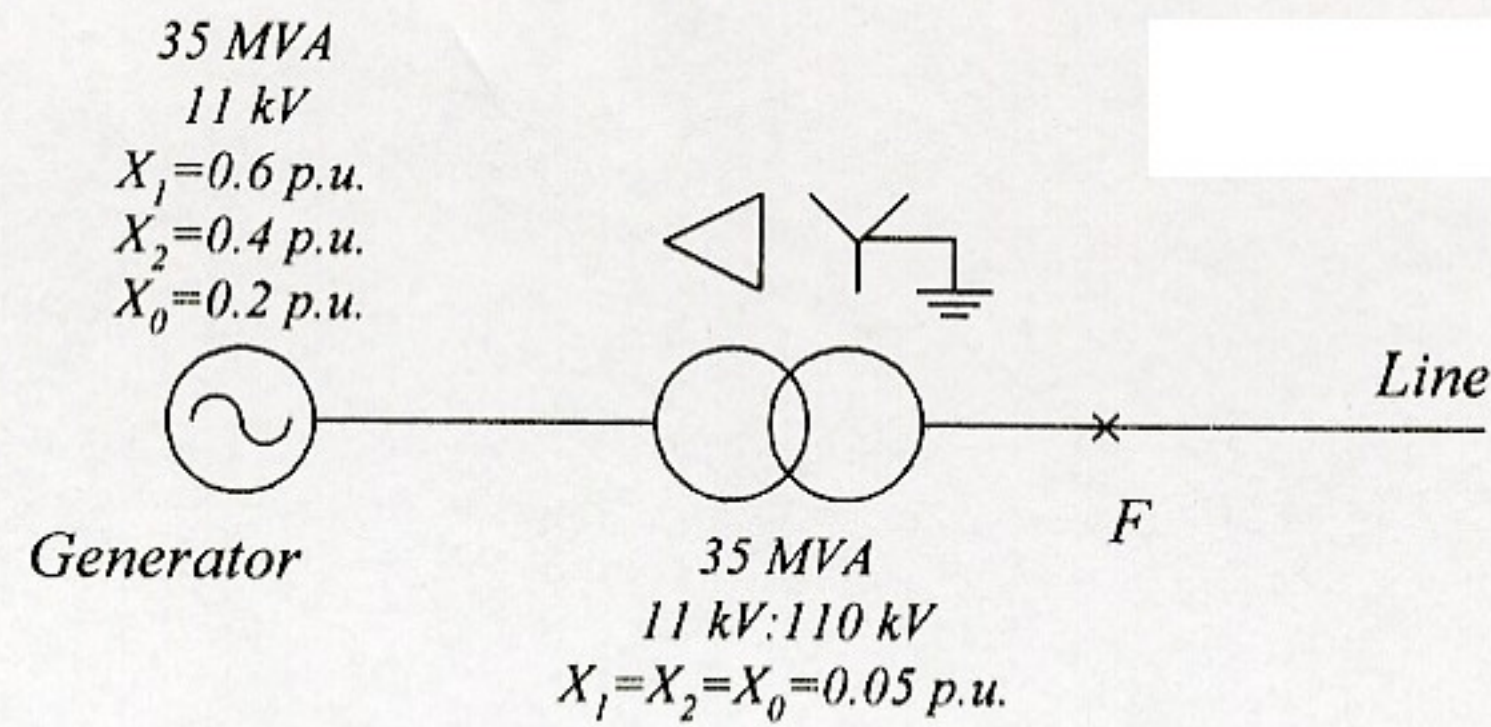


Fig. 2

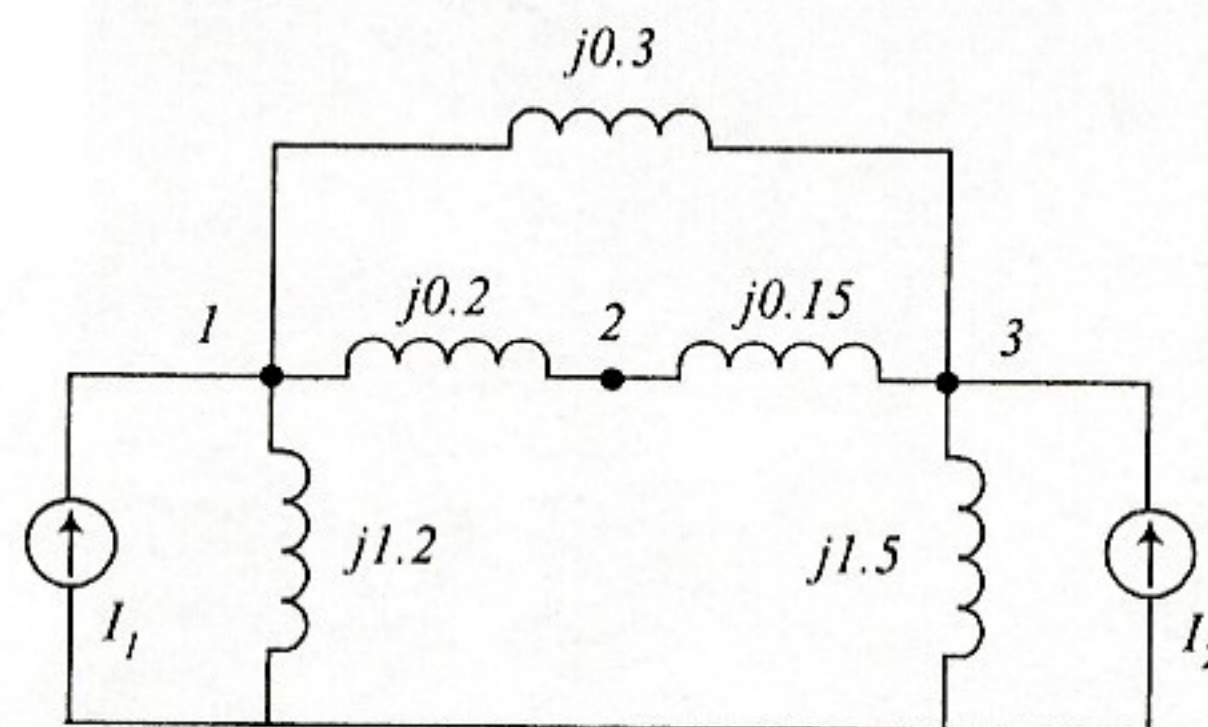


Fig. 3

6. (15%) An area of an interconnected power system has two fossil-fuel units operating on economic dispatch. The variable operating costs of these units are given by

$$C_1 = 10P_1 + 8 \times 10^{-3} P_1^2 \quad \$/\text{hr}$$

$$C_2 = 8P_2 + 9 \times 10^{-3} P_2^2 \quad \$/\text{hr}$$

where  $P_1$  and  $P_2$  are in megawatts. For a total output of 800 MW, calculate the output from each unit such that the total operating cost is minimal.





1. 以下為一些單位或組織之首頁的網址：

雲科大 <http://www.yuntech.???tw>

IBM <http://www.ibm.com>

IBM <http://www.ibm.com.tw>

工研院 <http://www.itri.???tw>

總統府 <http://www.president.???tw>

(a) 請問雲科大、工研院、總統府之首頁的網址中 $???$ ，應分別填入什麼？(5%)

(b) 請問 IBM 的兩種首頁網址有何差別？(3%)

(c) 請問 WWW (World Wide Web) 的主要用途何在？(4%)

(d) 請問 HTTP 與 WWW 的關係為何？(4%)

(e) 請問什麼是 HTML，它與 WWW 的關係為何？(4%)

2. 何謂同位元檢查 (parity check)？請舉一例說明其可能的應用。(7%)

3. (a) 請問  $(0.0101)_2$  之 2 的補數為何？(5%)

請利用 Two's complement representation 完成下列加法：

(b)  $(-25) + (15)$  (4%)

(c)  $40 + (-21)$  (4%)

4. (a) 什麼是數位影像(image)的像素(pixel)？(3%)

(b) 數位影像的彩色像素如何用數位方式以 RGB (red, green, blue) 三原色表達？(7%)

5. 請解釋下列有關計算機結構之名詞：

(a) interrupt-driven I/O (4%)

(b) isolated I/O (4%)

(c) memory-mapped I/O (4%)

(d) programmed I/O (4%)

6. 請解釋下列有關計算理論之名詞：

(a) NP problem (4%)

(b) NP-hard problem (4%)

7. 在作業系統中，

(a) 請用例子解釋何謂 starvation 和 deadlock？(4%)

(b) 請敘述產生 deadlock 的必要條件(necessary condition)。(4%)

(c) 請敘述 deadlock prevention 的方法。(4%)





8. 在記憶體管理中，

(a) 何謂分頁(paging)? (2%)

(b) 在決定頁之大小(page size)時，須考慮那些因素? (4%)

9. (a) 試分析圖(一)之C語言程式，並且試問當它執行後，它將印出的r值為何? (4%)

(b) 試分析圖(二)之C語言程式，並且試問當它執行後，它將印出的a和b值各為何? (4%)

```
#include <stdio.h>
int func(int, int);
int main()
{
    int x=2, y=4, z=6, r;
    r = func(func(++x, y--), --z);
    printf("r = %d\n", r);
    return 0;
}
```

func(int a, int b)

{ return a\*b; }

圖(一)

```
#include <stdio.h>
int main (void)
{
    int a, b;
    for(a=1, b=1; a<=8; a++)
    {
        if(b>22) break;
        if(b%3 == 1){ b+=3; continue; }
        b-=5;
    }
    printf("a = %d, b = %d\n", a, b);
    return 0;
}
```

圖(二)





1. Given a system described by the following dynamic equation

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t) \\ y(t) &= \begin{bmatrix} 0 & 3 \end{bmatrix} \mathbf{x}(t) = \mathbf{c}\mathbf{x}(t) \end{aligned}$$

- (a) Find  $\mathbf{x}(0)$  for zero input (i.e.  $u(t)=0$ ) and  $x(3) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ . (10%)

- (b) Let  $\Phi(t)$  be the state transition matrix of  $\mathbf{A}$ . Find  $[\Phi^{-1}(t)]^{20} = ?$  (8%)

2. Consider a system shown in Figure 1 for  $G(s) = \frac{k}{s(s+4)}$  and  $H(s) = 2$ . Find the range of  $k$  for which the system is (a) underdamped (b) overdamped?

(12%)

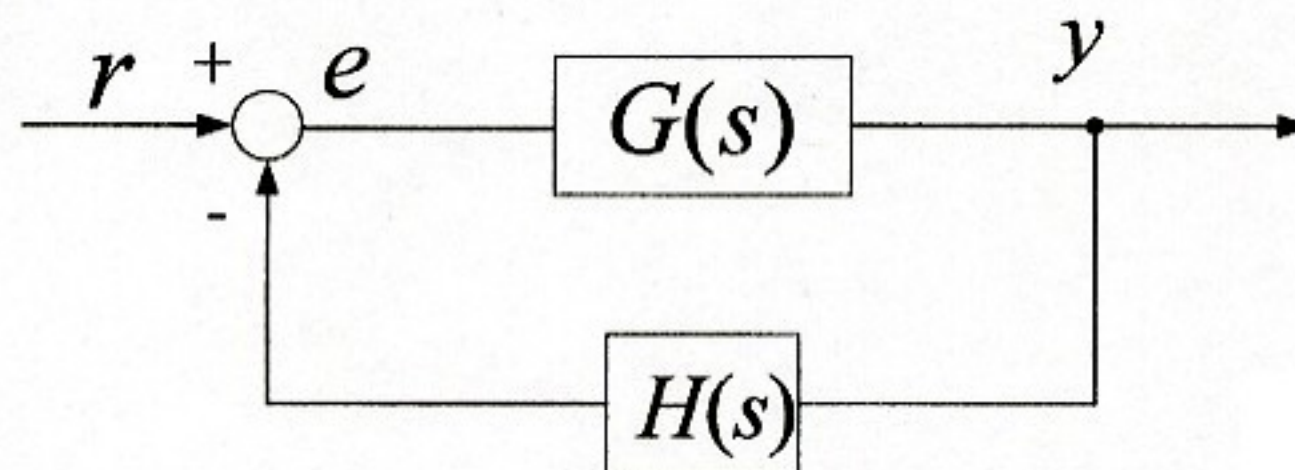


Figure 1

3. Consider a linear time-invariant system of the closed loop transfer function  $G_0(s) = \frac{Y(s)}{U(s)} = \frac{2s^2 + 2s + 9}{s^3 + 3s^2 + 2s + 9}$ , where  $u(t)$  and  $y(t)$  denote the input and the output, respectively. Find the steady state responses  $y(\infty)$  when  $u(t) = 1 + 4\sin(t)$ . (10%)

4. Given a system described by the following dynamic equation

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} 0 & 0 & 0 & 5 \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} u(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t) \\ y(t) &= \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}(t) = \mathbf{c}\mathbf{x}(t) \end{aligned}$$





Find the values of  $x_0$ ,  $x_1$ ,  $x_2$  and  $x_3$  for  $\mathbf{A}^6 = x_3 \mathbf{A}^3 + x_2 \mathbf{A}^2 + x_1 \mathbf{A} + x_0 \mathbf{I}$ ,

where  $\mathbf{I}$ :  $4 \times 4$  identity matrix.

(10%)

- 5 Consider a unity feedback system with the open-loop transfer function

$$G(s) = \frac{0.6}{s(s+1)(s+2)}$$

- (a) Plot the Nyquist diagram. (10%)
- (b) Using the Nyquist criterion, determine whether the closed-loop system is stable or not. (5%)
- (c) Find the gain margin. (5%)

- 6 Consider a unity feedback system with the open-loop transfer function

$$G(s) = \frac{16}{s(s+4)}$$

- (a) Determine the resonant frequency of the closed-loop system. (5%)
- (b) Determine the resonant peak value of the closed-loop system. (5%)
- (c) Determine the bandwidth of the closed-loop system. (5%)

7. A characteristic equation is  $(s+1)(s^2 + 5s + 6) + k = 0$ .

- (a) Find the breakaway points of the root locus and the corresponding value,  $k$ . (10%)
- (b) Find the stability region of  $k > 0$ . (5%)





1. Fig. 1 shows a direct ac voltage to ac current converter. Where  $q_{11} = 1$  when switch 1,1 is on and  $q_{11} = 0$  when switch 1,1 is off (The same is for  $q_{22}, q_{12}$ , and  $q_{21}$ ). If  $q_{11} = q_{22}$ ,  $q_{12} = q_{21}$ , and  $q_{12} = 1 - q_{11}$ , please derive the Fourier expression of  $v_{out}(t)$ .  $v_{out}(t) = [q_{11}q_{22} - q_{12}q_{21}]v_{in}(t)$ . (20%)

(The Fourier series of  $q(t)$  is  $D + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos(n\omega_{switch}t - n\omega_{switch}t_o)$ )

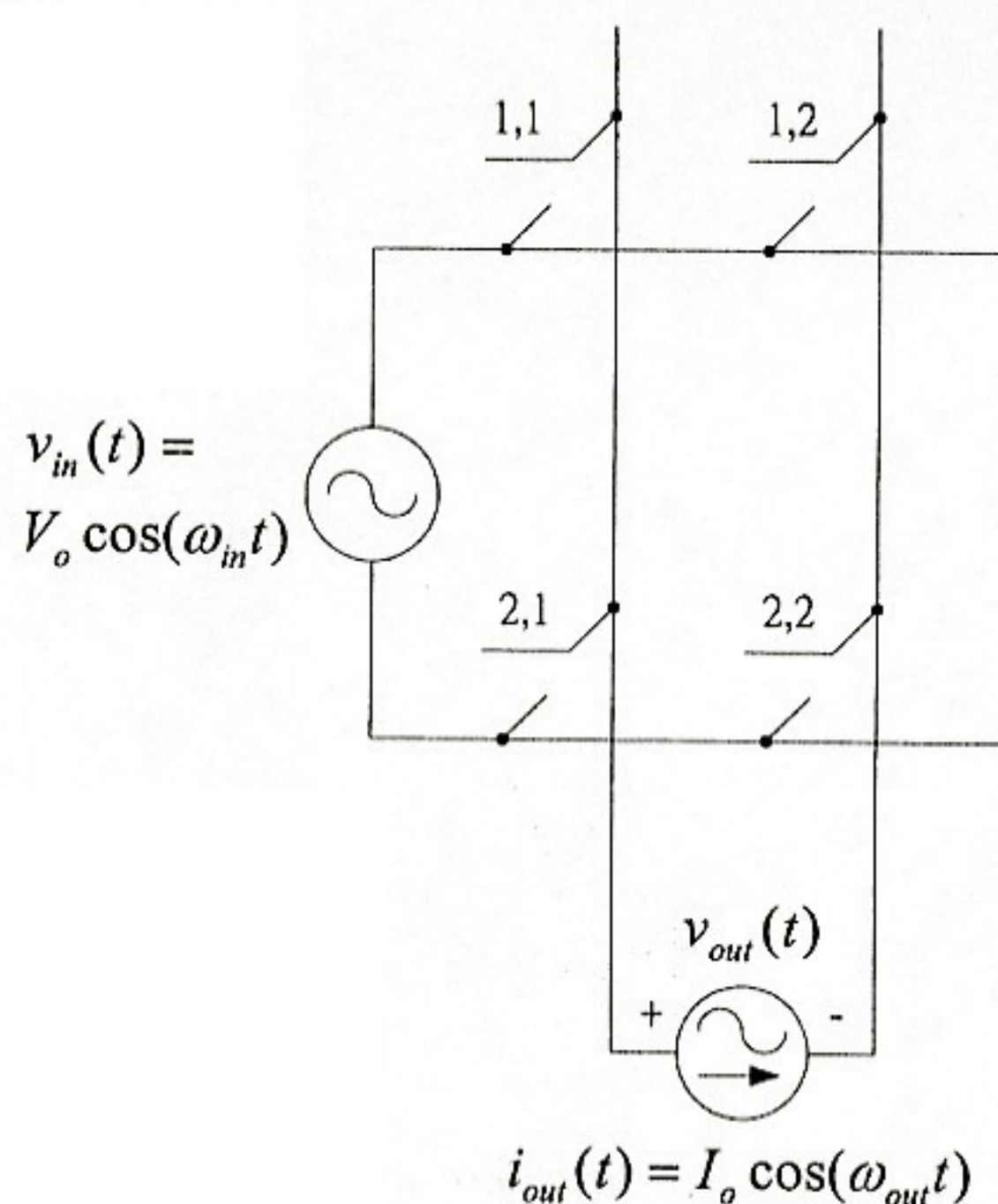


Fig. 1

2. Design a boost converter with 12V input and 200V output at up to 50W. The inductor current ripple  $\leq \pm 20\%$  and the output ripple should not exceed  $\pm 5\%$ . Switching frequency is 100 kHz. (25%)
3. Please derive the boundary condition between CCM and DCM for a buck-boost converter. (25%)
4. A rectifier bridge, shown in Fig. 2, supplies a series R-L load. Find the first three Fourier harmonics of the output current. (20%)



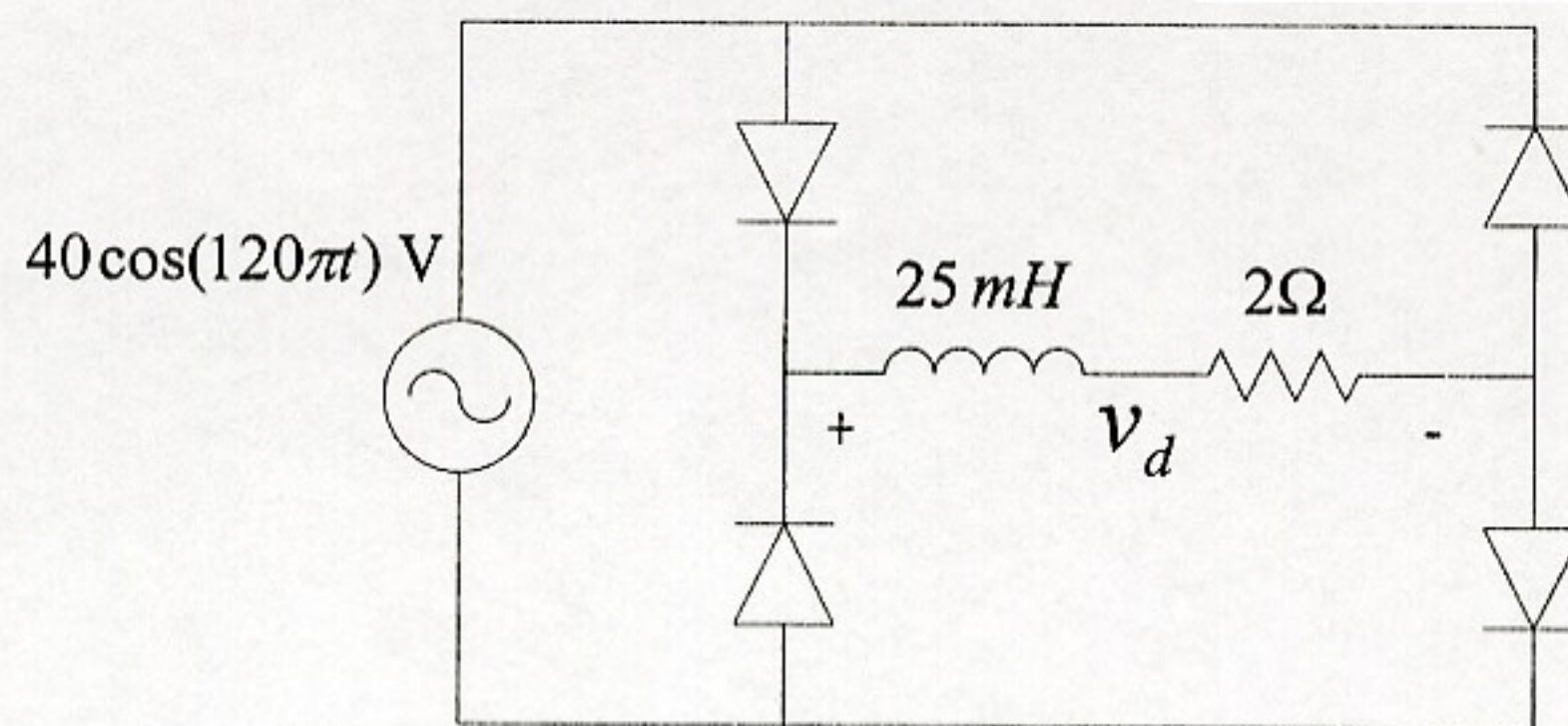


Fig. 2

5. Derive the steady state relationship between the input voltage and the output voltage of a buck converter (CCM). (10%)





1. The parameters of the transistors in the circuit in Figure 1 are  $V_{TND} = -1$  V,  $K_{nD} = 0.5$  mA/V<sup>2</sup> for transistor  $M_D$ , and  $V_{TNL} = +1$  V,  $K_{nL} = 30$   $\mu$ A/V<sup>2</sup> for transistor  $M_L$ . Assume  $\lambda = 0$  for both transistors.
- (a) Calculate the quiescent drain current  $I_{DQ}$  and the dc value of the output voltage. (10%)
- (b) Determine the small-signal voltage gain  $A_v = v_o / v_i$  about the  $Q$ -point. (10%)

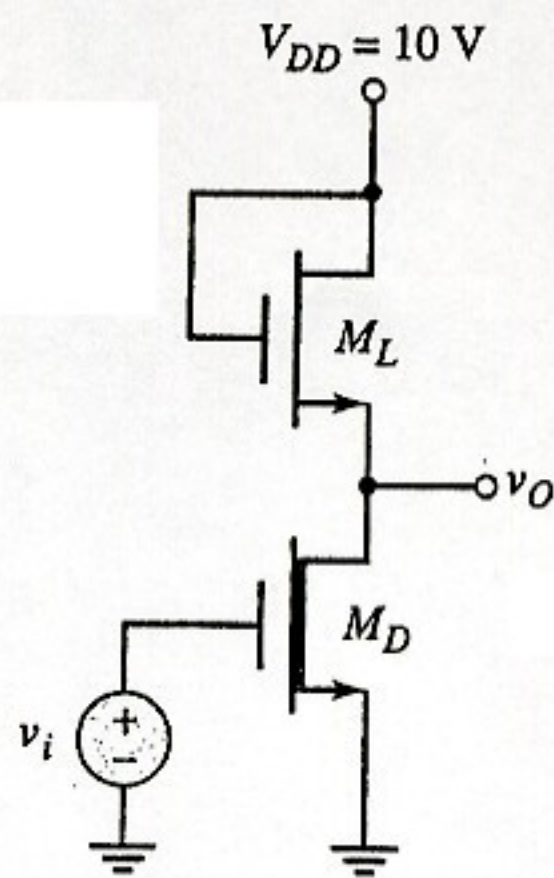


Figure 1

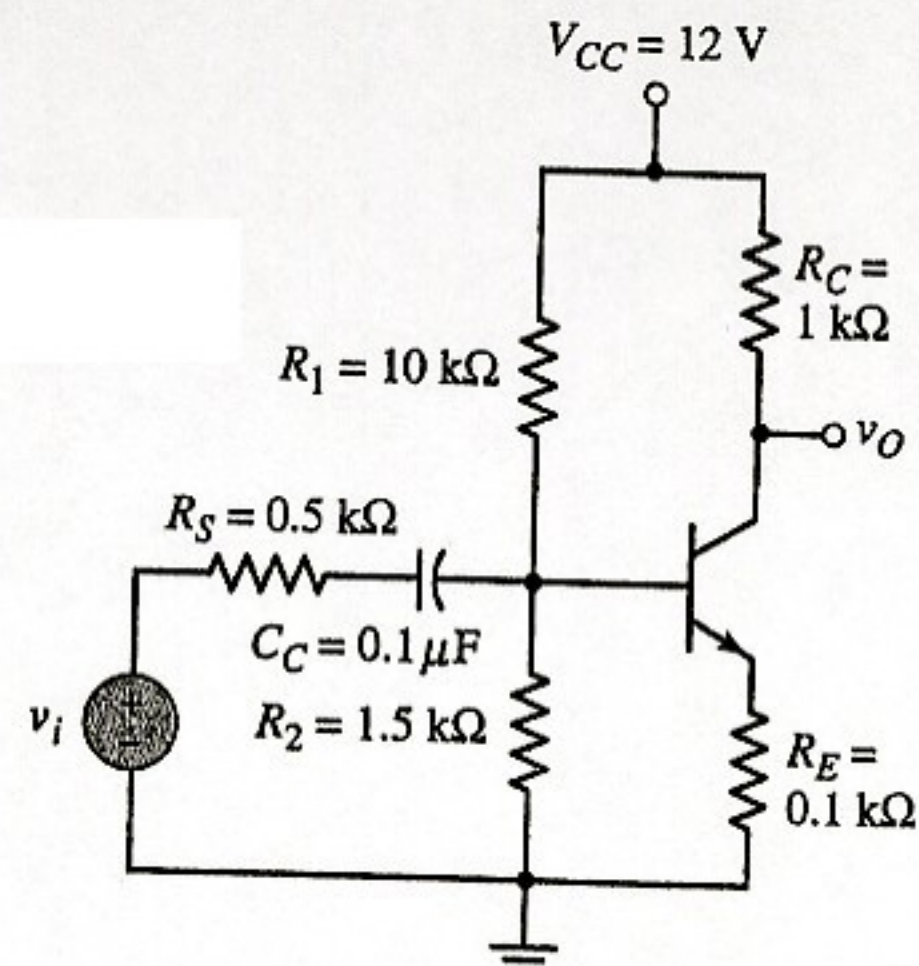


Figure 2

2. For the common-emitter circuit in Figure 2, the transistor parameters are  $\beta = 100$ ,  $V_{BE}(\text{on}) = 0.7$  V, and  $V_A = \infty$ .
- (a) Calculate the lower corner frequency. (5%)
- (b) Determine the midband voltage gain. (5%)
- (c) Sketch the Bode plot of the voltage gain magnitude. (10%)
3. The circuit in Figure 3 is an alternative configuration of a phase-shift oscillator. Assume that  $R_1 = R_2 = R_3 = R_{A1} = R_{A2} = R_{A3} = R$  and  $C_1 = C_2 = C_3 = C$ . Derive the expression for the frequency of oscillation. (20%)

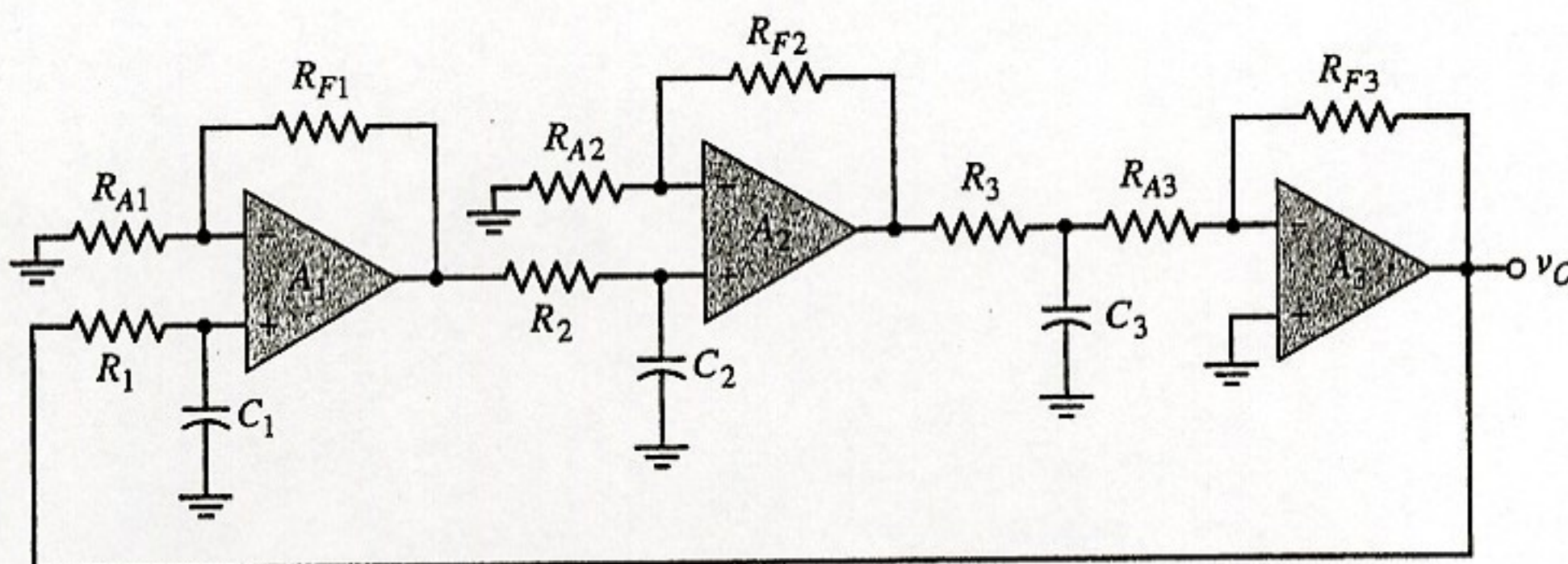


Figure 3





4. Consider the circuit in Figure 4. The output stage is a Darlington pair emitter-follower configuration. Assume that  $\beta = 100$  for all transistor and  $V_A = 100\text{ V}$  for  $Q_7$  and  $Q_{11}$ . Determine the output resistance  $R_o$ . (20%)

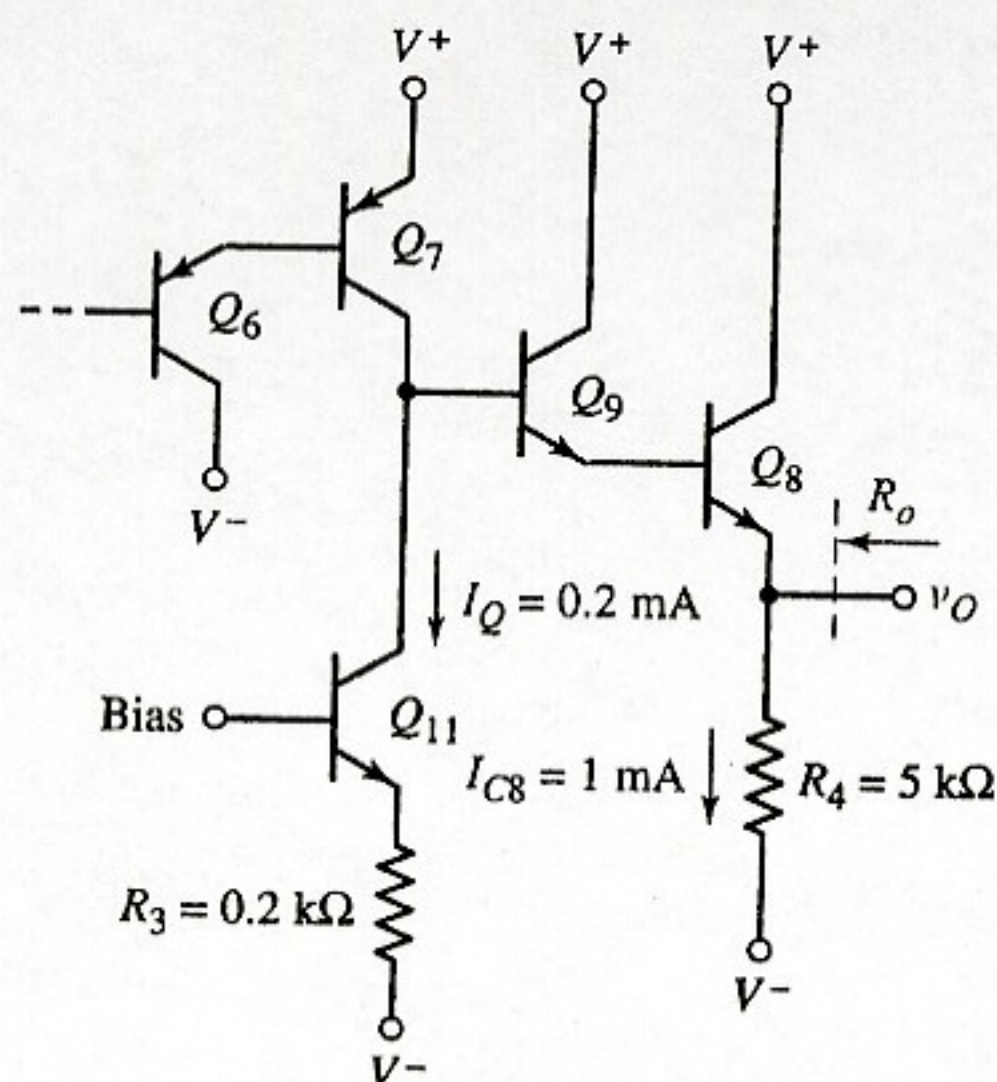


Figure 4

5. A 555 timer IC is connected as shown in Figure 5. Determine the range of oscillation frequency and the duty cycle. (20%)

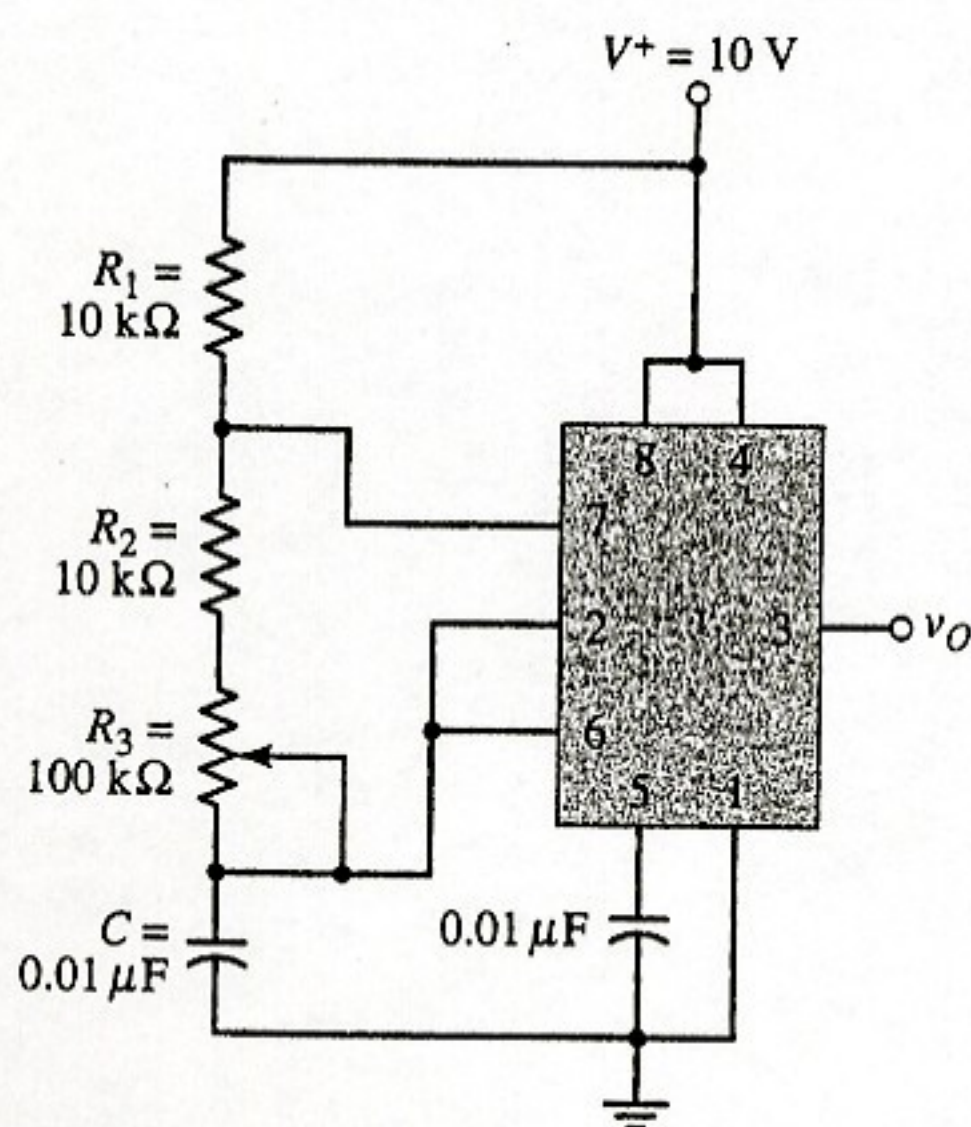


Figure 5





1. (15%) If  $A$  is a  $2 \times 2$  matrix with  $|A| = 4$ , use the properties of determinants to compute the following determinants.

(a)  $|3A|$     (b)  $|A^2|$     (c)  $|5A^T A^{-1}|$ , assuming  $A^{-1}$  exists

2. (15%) Determine the values of  $\lambda$  for which the following system of equations has nontrivial solutions. Find the solutions for each value of  $\lambda$ .

$$\begin{aligned} (5 - \lambda)x_1 + 4x_2 + 2x_3 &= 0 \\ 4x_1 + (5 - \lambda)x_2 + 2x_3 &= 0 \\ 2x_1 + 2x_2 + (2 - \lambda)x_3 &= 0 \end{aligned}$$

3. (20%) State (with a brief explanation) whether the following statements are **true** or **false**.

- (a) The vectors  $(1, 2), (-1, 3), (5, 2)$  are linearly dependent in  $\mathbf{R}^2$ .  
 (b) The vectors  $(1, 0, 0), (0, 2, 0), (1, 2, 0)$  span  $\mathbf{R}^3$ .  
 (c) The set,  $\{(1, 0, 2), (0, 1, -3)\}$ , is a basis for the subspace of  $\mathbf{R}^3$  consisting of vectors of the form  $(a, b, 2a - 3b)$ .  
 (d) Any set of two vectors can be used to generate a two-dimensional subspace of  $\mathbf{R}^3$ .

4. (15%)

- (a) (8%) Find a basis for the subspace  $W$  of  $\mathbf{R}^3$  spanned by

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 11 \\ 10 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix} \right\}.$$

- (b) (7%) Find a basis for  $\mathbf{R}^3$  that includes the vectors

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

5. (15%) Let  $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$  be the linear transformation defined by

$$T \left( \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \right) = \begin{bmatrix} a_1 + a_3 \\ a_1 + a_2 \\ a_2 - a_3 \end{bmatrix}$$

- (a) (8%) Find a basis for the **kernel space**,  $\ker(T)$ .  
 (b) (7%) Find a basis for the **range space**,  $\text{range}(T)$ .





6. (10%) Let  $V$  be the inner product space of all continuous real-valued functions on the interval  $[0, 1]$  with inner product defined by

$$\langle f, g \rangle = \int_0^1 f(t) g(t) dt, \quad \forall f, g \in V.$$

Let  $S = \{t, e^t\}$  be a basis for a subspace  $W$  of  $V$ . Apply the Gram-Schmidt process to find an orthonormal basis for  $W$ .

7. (10%) Let

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}.$$

Find an orthogonal matrix  $P$  and a diagonal matrix  $D$  such that  $P^T A P = D$ .





1. (25%) A three-phase circuit has a Y-connected load without neutral line as shown in Fig. 1. The phase voltages in RMS value are  $V_{an} = 220\angle 0^\circ$  V,  $V_{bn} = 220\angle -120^\circ$  V,  $V_{cn} = 220\angle 120^\circ$  V, and load impedances are  $z_1 = 6\angle 0^\circ \Omega$ ,  $z_2 = 6\angle 30^\circ \Omega$ ,  $z_3 = 5\angle 45^\circ \Omega$ , please find
- (10%) the line currents  $I_a$ ,  $I_b$  and  $I_c$  (in RMS);
  - (10%) the displacement voltage  $V_{n'n}$  (in RMS);
  - (5%) total complex power delivered to the three-phase load.

2. (15%) Find Thévenin equivalent of the circuit of Fig. 2 seen from terminals  $a$  and  $b$ .

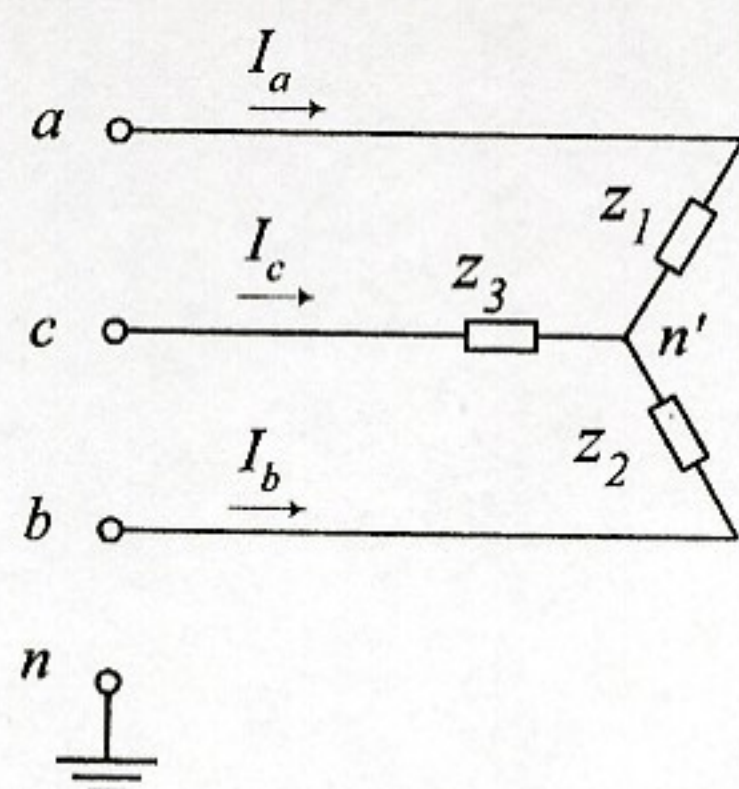


Fig. 1

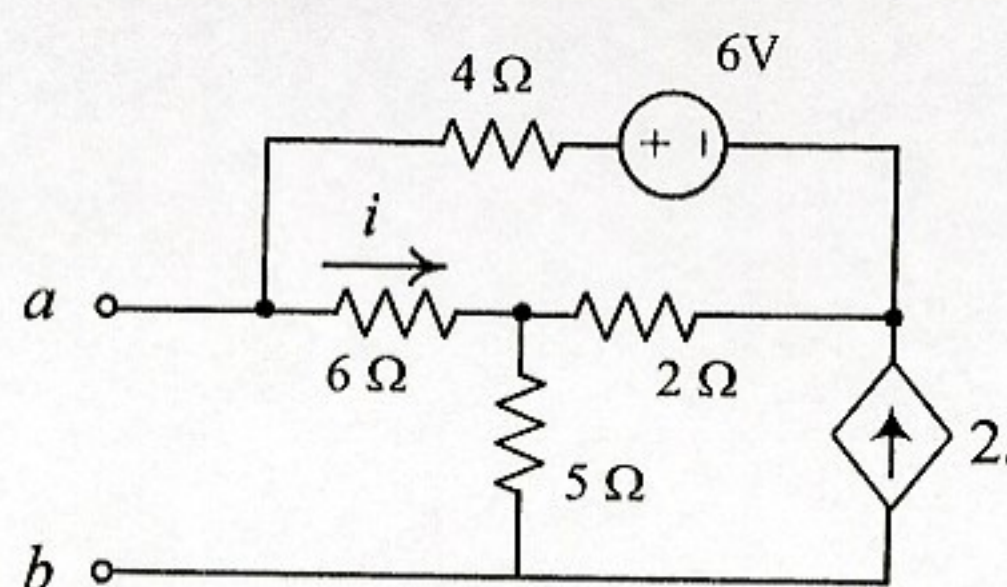


Fig. 2

3. (10%) A voltage of 10 V is applied at  $t=0$  to a series R-L-C circuit with  $R=5\Omega$ ,  $L=0.1$  H,  $C=500\mu$ F. All initial conditions are zero for  $t < 0$ . Find the transient voltage across the capacitor  $v_c(t)$  for  $t > 0$ .
4. (20%) Consider the network in Fig. 3.
- (5%) Obtain the voltage transfer function  $H_v(\omega)$  where  $V_1$  and  $V_2$  are the input voltage and the output voltage, respectively.
  - (15%) Let  $R_1 = 5\text{ k}\Omega$  and  $C = 10\text{ nF}$ . If  $H_v(\omega) = 0.8\angle 0^\circ$  at 15 kHz, calculate  $R_2$ ,  $L$ , and the bandwidth.

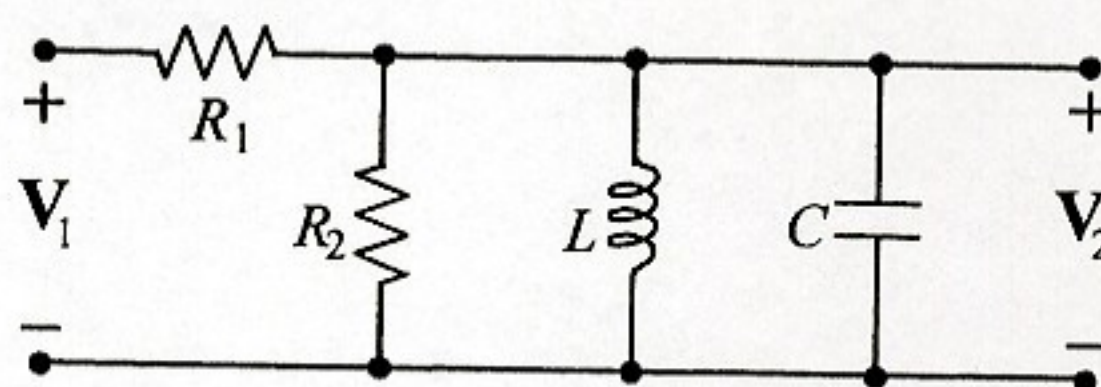


Fig. 3





5. (15%) Consider the two-port network in Fig. 4 with  $z$ -parameters of  $z_{11}$ ,  $z_{12}$ ,  $z_{21}$ , and  $z_{22}$ . This network is a non-reciprocal network.

(a) (5%) Show that the Thevenin equivalent voltage  $V_{Th}$  at the output terminals is

$$V_{Th} = \frac{z_{21}}{z_{11} + Z_s} V_s.$$

(b) (10%) Show that the Thevenin equivalent impedance  $Z_{Th}$  at the output terminals is

$$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s}.$$

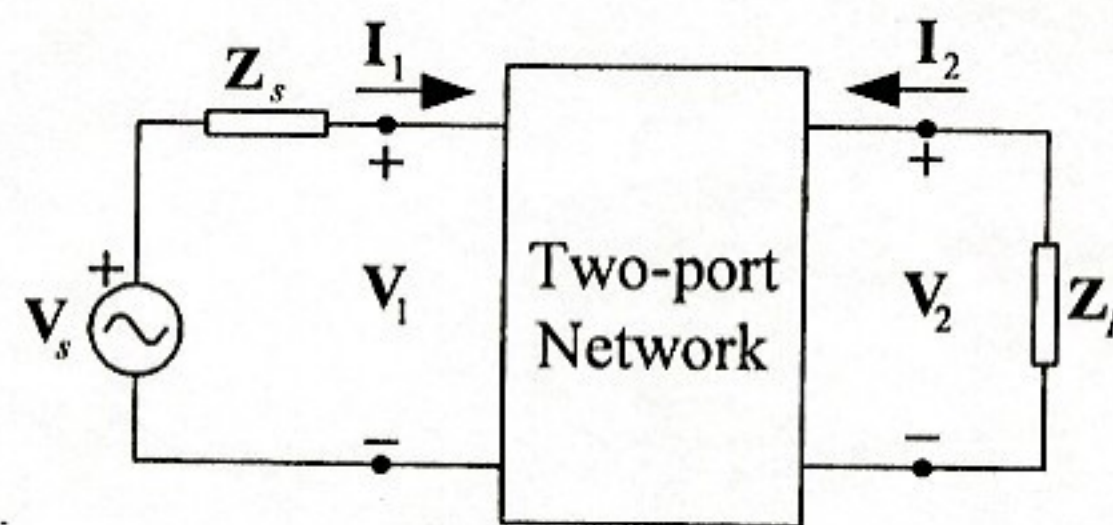


Fig. 4

6. (15%) Consider the circuit in Fig. 5.
- (a) (8%) Write the mesh-current equation in matrix form.
- (b) (7%) Find the voltage across the  $5\text{-}\Omega$  reactance with the polarity shown.

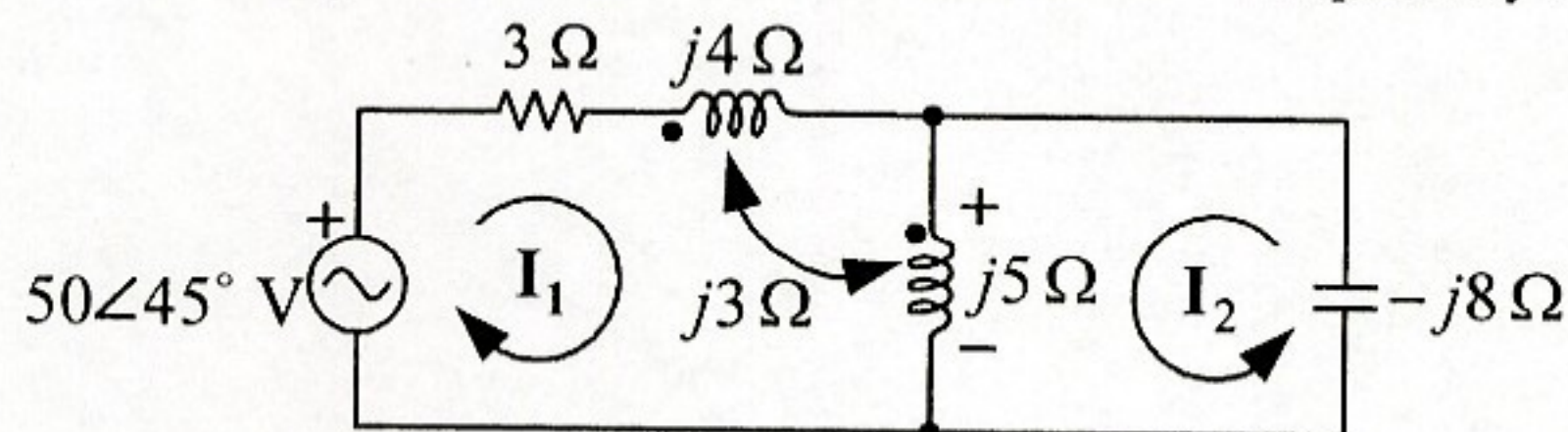


Fig. 5



**30% 第一題**

Three types of messages arrive at a message center. Ten percent of the messages are “high priority”, 40 percent are “normal priority”, and 50 percent are “low priority.”

10% (A) Find the probability that  $k$  out of  $N$  messages are not high priority.

10 % (B) Suppose messages arrive one at a time. Find the probability that  $k$  messages are received before a high-priority message arrives.

10% (C) Find the probability that out of 20 messages, 5 are high priority, 10 are normal priority, and 5 are low priority.

**20% 第二題**

假定某電腦公司生產的主機板使用的主機板使用的 256Mbytes DDR DRAM IC 是分別由 A、B 兩家公司供應(假定規格相同且相容)。根據市場調查，此二公司的不良率分別是 A: 0.01, B: 0.05。而進貨成本報價分別為 A:1500 元/每顆及 B:1000 元/每顆。公司須進貨 1000 顆，而每片主機板因 DRAM 產生的壞率被要求最多在 0.04。

10% (A) 如果你是採購經理，請問你該如何採購可達到良率要求卻是最低成本？

10% (B) 發現一顆 IC 是壞的，試問其為 B 公司的機率是多少？

**20% 第三題**

假定為解決台海問題，聯合國分別邀請中華民國政府與中華人民共和國政府各派一名不怕死的神槍手進行決鬥以決定獨立或被統一。中華民國政府派出的是神槍手 A，每回合射死對方的機會是  $P_1$ ，中華人民共和國政府派出的是神槍手 B，每回合射死對方的機會是  $P_2$ ，假定每回合決鬥是獨立的(受傷並不影響決鬥)並且命中率都保持同樣水準，比賽將繼續直到至少一方被射死為止，而且有可能兩人同時被對方射死。

5% (A) 決鬥將進行剛好 5 回合之機率是多少？

5% (B) 決鬥將進行剛好 5 回合並且中華民國政府的神射手 A 是存活之機率是多少？

5% (C) 不管決鬥將進行多少回合，中華民國政府的神射手 A 會存活之機率是多少？

5% (D) 假定比賽進行了 5 回合還分不出勝負，聯合國裁判問說還要進行剛好 5 回合才分出勝負之機率是多少？

**30% 第四題**

A factory has  $n$  machines of a certain types. Let  $p$  be the probability that a machine is working on any given day, and let  $N$  be the total number of machines working on a certain day. The time  $T$  required to manufacture an item is an exponentially distributed random variable with rate  $k\alpha$  if  $k$  machines are working.

15% (A) Find  $P[N = k, T \leq t]$  and  $P[T \leq t]$ .

15% (B) Find  $P[T \leq t]$  as  $t \rightarrow \infty$  and explain the result.





1. Given that the Laplace transform  $\mathcal{L}\left\{\frac{2}{t}[1 - \cos(t)]\right\} = \ln\left(\frac{s^2 + 1}{s^2}\right)$ ,  
please find the value of  $\mathcal{L}\left\{\frac{1}{t}[1 - \cos(2t)]\right\}$  (10%)
2. Find the inverse Laplace transform for the following function. (10%)  

$$\frac{se^{-s}}{(s+1)^2(s^2+2s+2)}$$
3. Find the general solution for the following differential equations. (30%)
  - (a)  $(D^4 + 5D^2 - 36)y(x) = 10e^{-2x} + 3\cos(3x)$ . (10%)
  - (b)  $(x^3D^3 + 3x^2D^2 + xD - 1)y(x) = 0$ . (10%)
  - (c)  $\frac{dy}{dx} = \frac{6xy - y^2}{3xy - 6x^2}$ . (10%)
4. Find the Fourier half cosine and Fourier half sine expansions of  $f(x)$  for  

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \end{cases}$$
 (15%)
5. Solve the following integral equation for the function  $f(x)$   

$$\int_0^\infty f(x) \sin(\omega x) dx = \begin{cases} 1, & 0 < \omega < 1 \\ 0, & \omega > 1 \end{cases}$$
 (10%)
6. Determine the polynomial  $y = a_0 + a_1x + a_2x^2$  whose graph passes through the points  $(x, y)$  of  $(1, 9)$ ,  $(2, 18)$ , and  $(3, 31)$ . (10%)
7. Consider the following linear equations  $\mathbf{Ax}=\mathbf{b}$ ,
 
$$\begin{cases} x_1 & - 2x_3 + x_4 = 4 \\ 3x_1 + x_2 - 5x_3 & = 8 \\ x_1 + 2x_2 & - 5x_4 = -4 \end{cases}$$

Write the solution in the form  $\mathbf{x}=\mathbf{x}_h + \mathbf{x}_p$ , where  $\mathbf{x}_h$  is the solution of  $\mathbf{Ax}=\mathbf{0}$  and  $\mathbf{x}_p$  is a particular solution of  $\mathbf{Ax}=\mathbf{b}$ . (8%)
8. Let  $A$  be an  $4 \times 4$  invertible matrix and  $\text{adj}(A)$  be the adjoint of  $A$ . Find the value  $x$  of the determinant  $|\text{adj}(A)| = |A|^x$ . (7%)