國立雲林科技大學
101 學年度碩士班暨碩士在職專班招生考試試題

系所：電機系、營建系
科目：工程數學(2)

本試題共 8 題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1 Solve the initial value problem: $\sin(x-y)+\cos(x-y)-\cos(x-y)y'=0$; $y(0)=7\pi/6$. (Hint: multiply the equation by an integrating facto to make the equation exact) (本題 10 分)

2 Find the general solution of the differential equation: $y''-y=2\sin^2(x)$. (DO NOT use the Laplace transform method) (本題 10 分)

3 Solve the initial value problem: $x^2y''-6y=8x^2$; $y(1)=1$, $y'(1)=0$. (本題 10 分)

4 Use the Laplace transform to solve the equation: $f(t)=\cos(t)+e^{-2t}\int_0^t f(\alpha)e^{2\alpha}d\alpha$. (本題 10 分)

5 Find the inverse Laplace transform of $F(s)=\dfrac{e^{-2s}}{s^2(s+3)^2}$. (本題 10 分)

6 Let $f(t)=1,\quad 0\le t\le\pi$, find the Fourier cosine series and the Fourier sine series of $f(t)$ on interval $[0,\pi]$. (本題 15 分)

7 Let $A=\begin{bmatrix}8 & -4 & 3\\ 1 & 5 & -1\\ -2 & 6 & 1\end{bmatrix}$, $X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$, and $B=\begin{bmatrix}0\\ -5\\ -4\end{bmatrix}$: (本題 15 分)

(1) find the determinant ($|A|$) of the matrix A, and find the solution of $AX=B$ by Cramer's rule. (本小題 8 分)

(2) Find the inverse matrix (A^{-1}) of the matrix A, and find the solution of $AX=B$ by $X=A^{-1}B$. (本小題 7 分)

8 Let $A=\begin{bmatrix}7 & -1\\ 1 & 5\end{bmatrix}$, $X=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, and $X(t=0)=X(0)=\begin{bmatrix}5\\ 3\end{bmatrix}$: (本題 20 分)

(1) find the eigenvalues and eigenvectors of A. (本小題 5 分)

(2) find a fundamental matrix ($\Omega(t)$) for the systems of linear differential equations, $X'=AX$. (本小題 5 分)

(3) find the general solution of the system $X'=AX$. (本小題 5 分)

(4) solve the initial value problem of $X'=AX$ with $X(0)$. (本小題 5 分)

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系所：電機系
科目：線性代數(1)

(1) Determine the **values** of k such that the following equations $\begin{cases} 4x+ky=6 \\ kx+y=-3 \end{cases}$

(i) the equation has **no solution.** (4%)

(ii) the equation has **exact one solution.** (4%)

(iii) the equation has **an infinite number of solutions.** (4%)

(2) Determine the polynomial $p(x)=a_0+a_1x+a_2x^2$ whose graph passes the given points $(1,2),(2,0),(3,4)$. (12%)

(3) Solve the following linear system $\mathbf{Ax}=\mathbf{b}$ with LU-Factorization of

$$\begin{aligned} 2x_1+x_2 \quad &= 1 \\ x_2-x_3 &= 2 \\ -2x_1+x_2+x_3 &= -6 \end{aligned}$$

(i) Find the LU-Factorization of the coefficient matrix $\mathbf{A}$, where diagonal elements of L are 1. (8%)

(ii) From (i), solving $\mathbf{y}$ of the lower triangular system $L\mathbf{y}=\mathbf{b}$, where $\mathbf{y}=U\mathbf{x}$. (4%)

(iii) From (i) and (ii), solving $\mathbf{x}$ of the upper triangular system $U\mathbf{x}=\mathbf{y}$. (4%)

(4) Express the vector $\mathbf{b}$ as a linear combination of the columns of $\mathbf{A}$.

$$\mathbf{A}=\begin{bmatrix} 1 & 1 & -5 \\ 1 & 0 & -1 \\ 2 & -1 & -1 \end{bmatrix}, \quad \mathbf{b}=\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}.$$ (10%)

(5) Let $\boldsymbol{T}$ be the triangle with vertices at $(x_1,y_1),(x_2,y_2),(x_3,y_3)$. Show that

$$\{\text{area of } \boldsymbol{T}\}=\frac{1}{2}\left|\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}\right|$$ (10%)

(6) Find the area of the region $\boldsymbol{E}$ bounded by the ellipse whose equation is

$$\frac{x^2}{4}+\frac{y^2}{9}=1.$$ (10%)

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系所：電機系
科目：線性代數(1)

(7) (a) Find rank **A** and dim Null **A**. (8%)

(b) Find bases for the row space, the column space, and the null space of the matrix **A**.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}$$ (12%)

(8) Compute $\mathbf{A}^{10}$ where $\mathbf{A} = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$. (Hint: utilize $\mathbf{A} = \mathbf{PDP}^{-1}$) (10%)

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101學年度碩士班暨碩士在職專班招生考試試題

系所：電機系
科目：線性代數與機率學

1. 兩向量 **a** 與 **b** 其夾角為θ，請問當θ的角度範圍各為多少時，這兩向量的內積(Inner product)之值分別為 **a** • **b**>0, **a** • **b**=0, **a** • **b**<0? (10 分)

2. 考慮三維空間中一四面體(Tetrahedron)之三個相鄰邊緣分別以三個向量 **a, b, c** 來表示，若 **a**=i+3k, **b**=4i+6j+2k, **c**=3i+3j-6k, 求此四面體之體積。(10 分)

3. (a)求下列矩陣之奇異值(Eigen values)與其分別對之應奇異向量(Eigen vectors)，在什麼條件下矩陣只有一個奇異值？ (15 分)

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

(b)請計算A^9若矩陣A表示如下。 (15 分)

$$A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$

4. 在一場考試中，考題包括 10 個選擇題，每題答對得 10 分，答錯倒扣 3 分。每題有四個答案選項，其中只有一個是正確的。如果經倒扣後總分低於零分，則以零分計。假設某位考生在考試中的習慣是「每一題都用猜的」。
(a) 這位考生得分的期望值 (expected value) 是多少？ (8 分)
(b) 這位考生考試及格 (得分 ≥ 60) 的機率是多少？ (4 分)
(c) 假設考試不及格的話，考生可要求重考，直到及格為止。則他平均需要考幾次才能及格？ {令(b)小題的答案為p，本小題答案請用p表示} (4 分)

5. 由 −1 到 +3 之間任意挑選一個數字 x, 另外由 −3 到 +1 之間任意挑選另一個數字 y, 計算下列事件的發生機率：
(a) $x^2 < 0.25$; (4 分)
(b) $\max(x, y) > 0$; (4 分)
(c) $|x-y| < 1$; (4 分)
(d) $xy > 0$. (4 分)

6. 已知 X 與 Y 為互相獨立之高斯分佈 (Gaussian distributed) 隨機變數，且兩者之平均值及標準差相同，各為 $\mu_X = \mu_Y = 0$, $\sigma_X = \sigma_Y = 1$.
(a) 寫出 X 與 Y 的聯合機率密度函數 (joint pdf); (5 分)
(b) 令 $T = \dfrac{1}{\sqrt{X^2+Y^2}}$, 寫出 T 的累積分配函數 (cumulative distribution function, CDF); (8 分)
(c) 寫出 T 的機率密度函數 (pdf)。 (5 分)

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系所：電機系
科目：電路學與電力系統

1. Three identical impedances $Z_1 = Z_2 = Z_3 = 30\angle 30°\ \Omega$ are connected in Δ to form a balanced three-phase load as shown in **Fig. 1**. This load is supplied from a balanced three-phase voltage with $V_{an} = 100\angle 0°\ V$, $V_{bn} = 100\angle(-120°)\ V$ and $V_{cn} = 100\angle(120°)\ V$. Find (a) the reading values of Watt meters P_A and P_B；(b) total active power delivered by the source; (c) total reactive power delivered by the source. (30%)

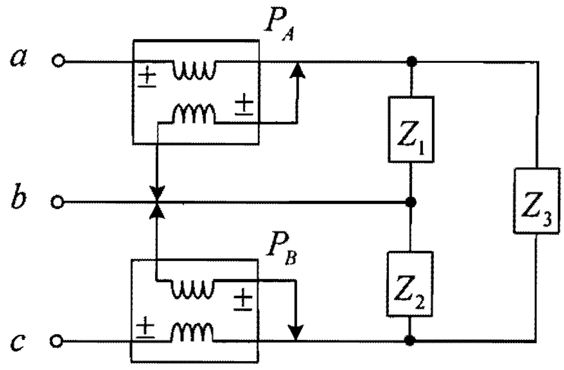

Fig. 1

2. In the circuit shown in Fig. 2, find (a) the voltage time function $v(t)$; (b) the active power and reactive power delivered by current source $i_1(t)$. (20%)

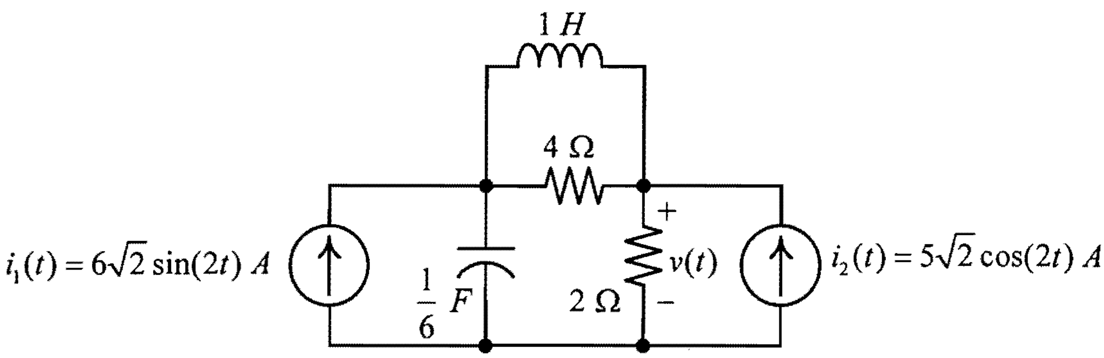

Fig. 2

3. An industrial load consisting of a bank of induction motors consumes 40 kW at a power factor of 0.8 lagging from a 220 V, 60 Hz, single-phase source. By placing a bank of capacitors in parallel with the load, the resultant power factor is to be raised to 0.95 lagging. Find the net capacitance of the capacitor bank in μF that is required. (10%)

4. A single-phase 100 kVA, 2400/240 V, 60 Hz distribution transformer is used as a step-down transformer. The load, which is connected to the 240 V secondary winding, absorbs 80 kVA at 0.8 power factor leading and at 230 V. Assuming an ideal transformer, calculate the following: (a) primary voltage, (b) load impedance, (c) load impedance referred to the primary, and (d) the real and reactive power supplied to the primary winding. (15%)

5. A 60 Hz single-phase, two-wire overhead line has solid cylindrical copper conductors with 1.5 cm diameter. The conductors are arranged in a horizontal configuration with 50 cm spacing. The line length is 20 km. For the single-phase line, calculate: (a) the total inductance in H and the total inductive reactance in Ω, (b) the line-to-line capacitance in F and the line-to-line admittance in S. (15%)

6. A 20 km, 34.5 kV, 60 Hz three-phase line has a positive-sequence series impedance $z = 0.19 + j0.34\,\Omega/\text{km}$. The load at the receiving end absorbs 10 MVA at 0.9 power factor lagging and at 33 kV. Assuming a short line, calculate: (a) the *ABCD* parameters, (b) the sending-end voltage. (10%)

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系所：電機系
科目：自動控制(2)

1. (15%) Given the unity feedback system of Figure 1,

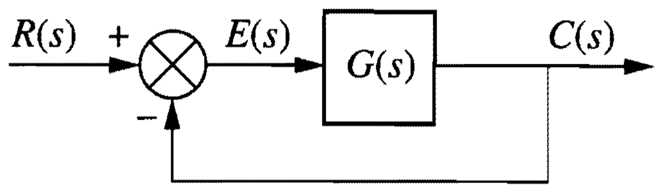

Figure 1

where

$$G(s) = \frac{K(s+\alpha)}{(s+\beta)^2}$$

is to be designed to meet the following specifications: steady-state error = 0.25 for unity step input; damping ratio = $\frac{1}{\sqrt{2}}$; natural frequency = $\sqrt{16}$. Find K, α and β.

2. (20%) Given the state space of the system represented below, where $u(t)$ is the unit step.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

a. (8%) Determine the state-transition matrix, e^{At}.

b. (5%) Determine the characteristic equation.

c. (7%) Find the output $y(t)$.

3. (15%) Find the transfer function, $G(s) = \frac{V_o(s)}{V_i(s)}$, for the network shown in Figure 2.

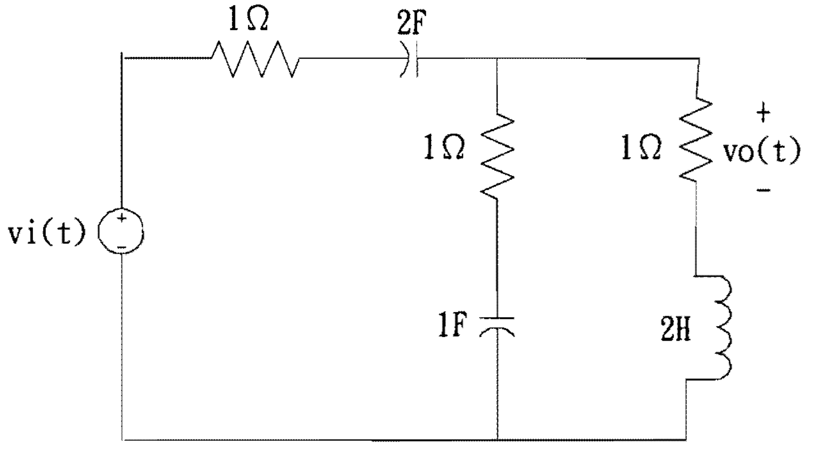

Figure 2

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系所：電機系
科目：自動控制(2)

4. (15%) For the system shown in Figure 3, find the following:

   a. (5%) The closed-loop transfer function, $T(s) = \frac{C(s)}{R(s)}$.

   b. (5%) The system type.

   c. (5%) The steady-state error, $r(\infty) - c(\infty)$, for the following test inputs: $15u(t)$, $15tu(t)$, and $15t^2u(t)$, respectively.

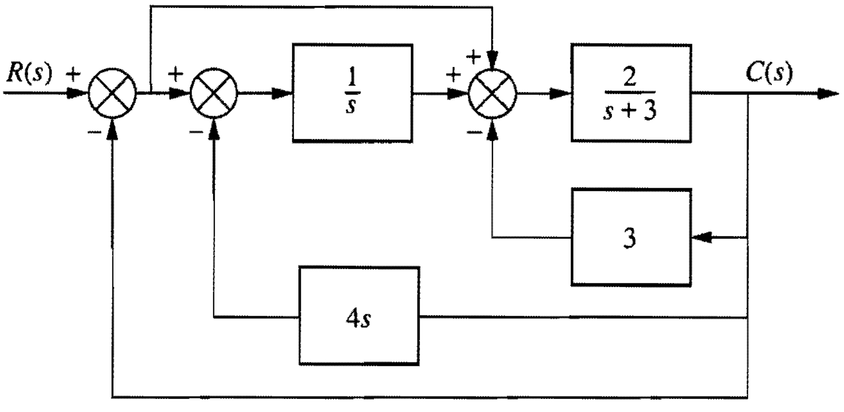

Figure 3

5. (20%) Given the unity feedback system of Figure 1, where

$$G(s) = \frac{K(s+1)}{s(s+2)(s+3)(s+5)}$$

do the following:

   a. (8%) Sketch the root locus

   b. (5%) Find the value of gain that will make the system marginally stable.

   c. (7%) Find the value of gain for which the closed-loop transfer function will have a pole on the real axis at -0.5.

6. (15%) Given the unity-feedback system shown in Figure 1, where

$$G(s) = \frac{50K}{s(s+3)(s+6)}$$

   a. (8%) For $K = 1$, sketch the Nyquist diagram and determine if the system is stable.

   b. (7%) Using the Nyquist criterion, find the range of K for stability of the system.

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系所：電機系
科目：計算機概論(1)

1. (a) (5%)在網際網路中，何謂 URL？
   (b) (5%)試舉出一個 URL 的範例。

2. (a) (3%)何謂 Multitasking (多工)？
   (b) (6%)從 CPU 的角度來看，作業系統如何處理多工的問題？
   (c) (6%)從記憶體的角度來看，作業系統如何處理多工的問題？

3. (a) (8%)請說明如何建構一個資料庫系統？ (提示：包含資料表 Table…等等)
   (b) (7%)何謂 SQL？試舉一例說明其基本語法。

4. (10%)請利用 802.11 CSMA/CA 技術，說明無線上網的基本原理?

5. (a) (5%)請說明 CPU 架構中，何謂范紐曼(von Neumann)架構？
   (b) (5%)何謂哈佛(Harvard)架構？
   (c) (5%)其優缺點各為何?

6. (20%)Fibonacci sequence (費式數列) 定義為 f(n) =f(n-1)+f(n-2), 同時 f(0)=1, f(1)=1 且 n 為正整數，試寫 C 語言函式：n 為輸入值，回傳值為費氏數列數值 f(n)，分別採用遞迴寫法(10%)與非遞迴的寫法(10%)。

7. 以 IEEE754 格式表示浮點數(floating)
   (a) (8%)計算 1.5625 的二進位格式。
   (b) (7%)計算 | 1 | 0 1 1 1 1 1 0 1 | 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 的十進位數值。

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系所：電機系
科目：電子學(1)

1. Figure 1 shows a typical output characteristic of an *npn* bipolar transistor biased at a given base current. Two operating points (Q_A, Q_B) are chosen.
   (a) Please compare the magnitude of output resistance (r_{OA} vs. r_{OB}) for the two operating points (Q_A and Q_B). (3%)
   (b) Please compare the magnitude of the transconductance (g_{mA} vs. g_{mB}) for the two operating points (Q_A and Q_B). (3%)

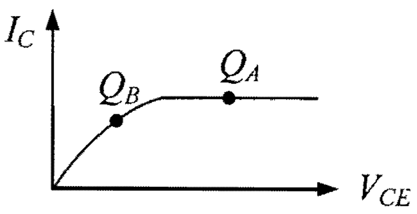


Figure 1

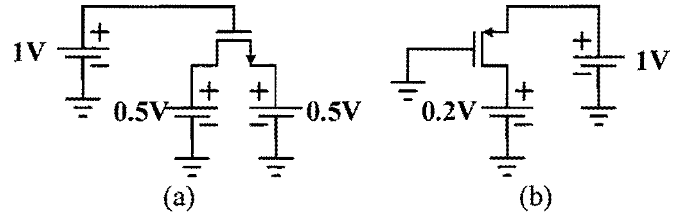


Figure 2

2. Please analyze and denote the operating regions (triode, saturation or cut-off) of the MOS circuits depicted in Fig. 2. Assume V_{THN}=| V_{THP} |=0.4V (4%)
3. Two outputs of the amplifier are specified in Fig. 3. Derive the voltage gain, v_{out1}/v_{in} and v_{out2}/v_{in} , respectively. Set λ=0 for M_1 and v_b as a fixed voltage. (10%)

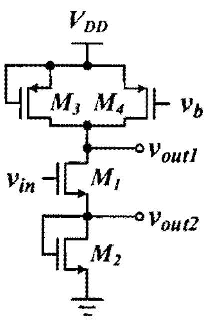


Figure 3

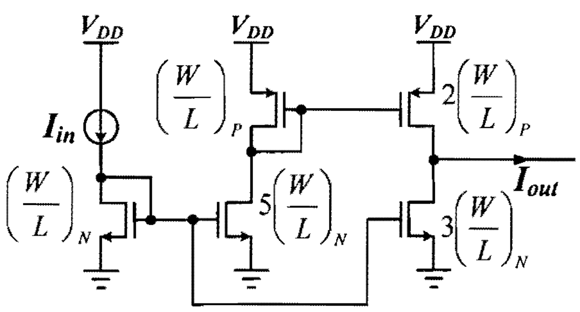


Figure 4

4. Assume all of the transistors drawn in Fig.4 operate in saturation region. Please calculate the current gain (=I_{out}/I_{in}). (10%)
5. Please identify the amplifier types in Fig. 5 and also denote the ideal requirements of the input/output impedances (R_{in}, R_{out}). The amplifier can be either of the following type : voltage, current, transimpedance and transconductance. (20%)

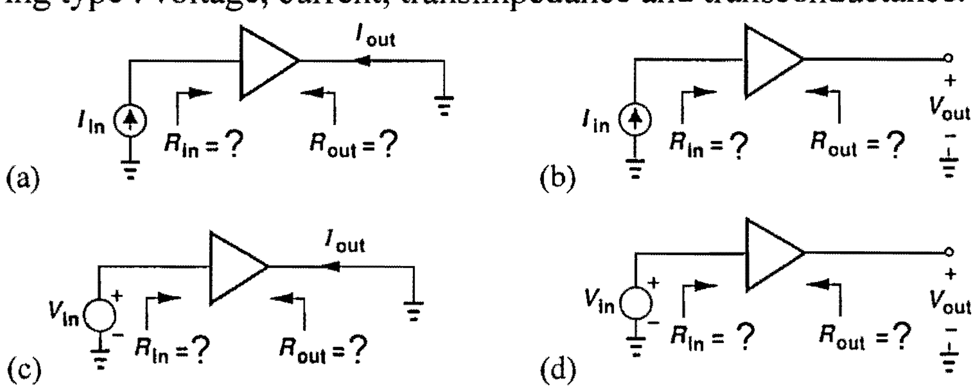


Figure 5

6. Draw the small-signal equivalent circuit for the amplifier shown in Fig.6. (10%)

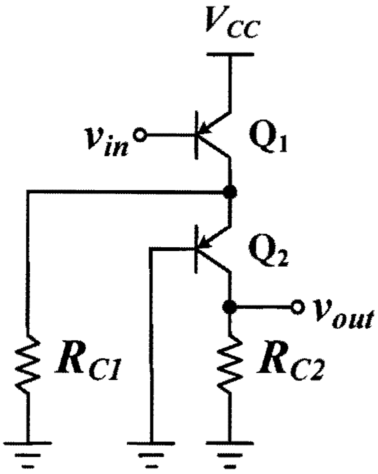


Figure 6

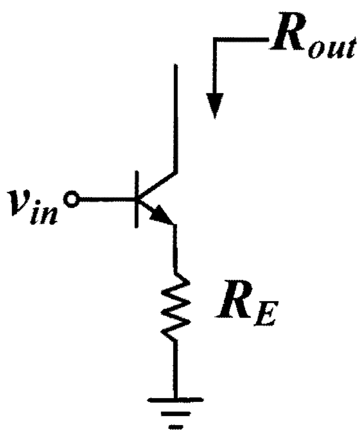


Figure 7

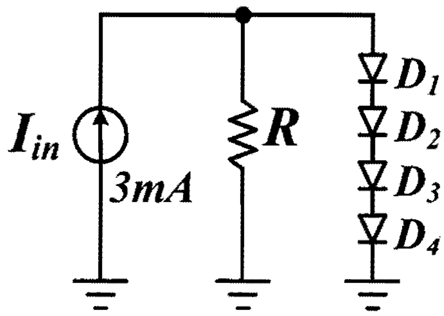
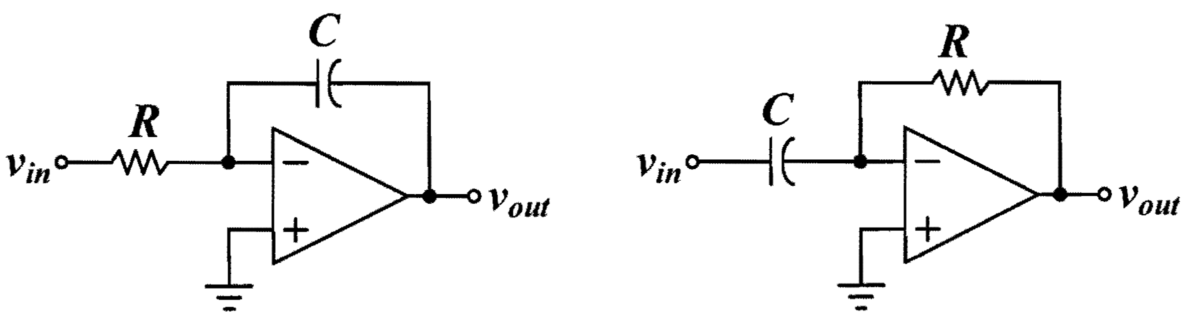


Figure 8

7. For the degenerated circuit depicted in Fig. 7, please perform the small signal analysis to derive the exact output resistance (R_{out}) and its simplified form detailed explanation. (15%)

8. For the circuit shown in Fig. 8, the current flowing through the resistor is 2.5 mA. Please calculate the voltage across the resistor and the required value of the resistor. Assume $I_S = 5 \times 10^{-16}$ A for each diode at room temperature. (10%)

9. For the two operational amplifier circuits shown in Fig. 9, please answer the following questions : (15%)
   (1) Derive the transfer functions and their pole or zero by assuming the gain of the operational amplifier is infinite. What are the main purposes of these two circuits?
   (2) If the gain of the operational amplifier is finite to be A_0, please recalculate the transfer functions and their pole or zero.



(a) (b)

Figure 9

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系所：電機系
科目：信號與系統

There are totally 7 questions, totally 100 points. Please answer the following questions in order, otherwise no score will be considered.

1. Determine the fundamental period of the signals below:
   (a) (3%) Determine the fundamental period of the signal $2\sin(3t+2)$?
   (b) (3%) Determine the fundamental period of the signal $3\cos(4t+3)$?
   (c) (4%) Determine the fundamental period of the signal $[2\sin(3t+2)-3\cos(4t+3)]$?

2. Consider the feedback system as Figure 1. Assume that $y[n]=0$ for $n<0$:
   (a) (5%) Sketch the function of the output $y[n]$ when $x[n]=\delta[n]$?
   (b) (5%) Sketch the function of the output $y[n]$ when $x[n]=u[n]$?

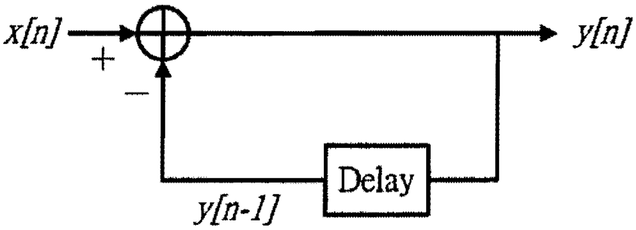

Figure 1

3. Let $x(t)$ be the rectangular pulse shown in Figure 2(a), and let $h(t)$ be the impulse train depicted in Figure 2(b). That is

$$h(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT)$$

Determine and sketch $y(t)=x(t)*h(t)$ when T is equal to the following value:

(a) (5%) T =4? (b) (5%) T =2? (c) (5%) T =3/2? (d) (5%) T =1?

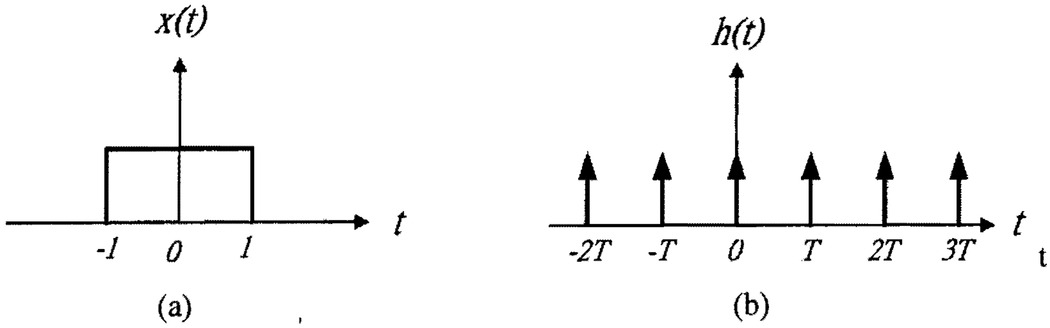

Figure 2

4. Let $x(t)$ be a periodic signal with fundamental period T and Fourier series coefficients a_k. That is, $a_k=\frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$ and $x(t)=\sum_{k=-\infty}^{\infty}a_k e^{jk(2\pi/T)t}$. Please derive the Fourier series coefficients of the following signals in terms of a_k:

(a) (5%) $x(5t-t_0)$? (b) (5%) $\frac{d^2x(t)}{dt^2}$?

國立雲林科技大學 系所：電機系
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5. (16%) AM Modulation. Given a voice signal s(t) whose Fourier Transform (spectrum) S(ω)=10 between the range ($-\omega_1, \omega_1$) and S(ω)=0 otherwise. Let p(t)= $\cos\omega_0 t$ be the modulation signal, assume $\omega_0 >> \omega_m$

(a) Sketch S(ω).

(b) Sketch $P(\omega)$, the Fourier Transform of p(t) (Hint: $P(\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$)

(c) The transmitted signal r(t)=s(t)p(t), find and sketch R(ω), the Fourier Transform of r(t).

(d) The demodulated signal g(t)=r(t)p(t), find and sketch G(ω), the Fourier Transform of g(t). How to recover s(t) from g(t)?

(Hint: Multiplication in time domain corresponds to convolution in the frequency domain)

6. (16%) Digitization is the process to obtain a discrete time signal x[n] from a continuous time signal x(t).

(a) Name the two major steps in digitization.

(b) Why do digital processing of continuous time signals become standard in most applications?

(c) Given a signal x(t) with non-zero frequency contents between ($-2\pi \cdot 4000, 2\pi \cdot 4000$). Accordingly to the sampling theorem, what is the minimal sampling period T so that x(t) can be reconstructed from x[n] without aliasing?

7. (18%) Aliasing as Figure 3

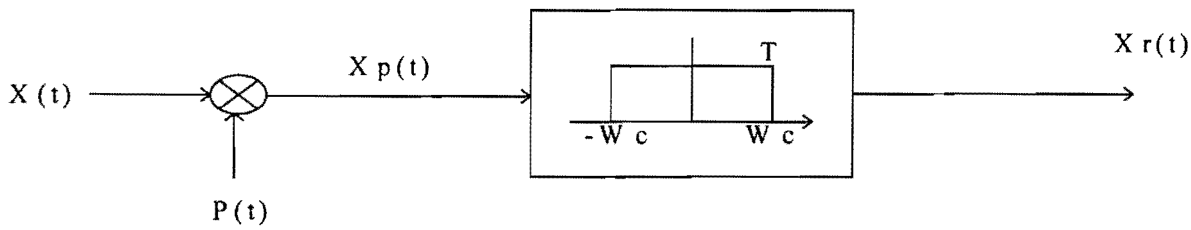

Figure 3

where:

$$\begin{cases} X(t) = \cos\omega_0 t, \quad Sampling\ \ function\ \ P(t) = \sum_{k=-\infty}^{k=+\infty} \delta(t - kT) \\ Xp(t) = X(t)P(t), \quad \omega_C = \dfrac{\omega_S}{2}, \quad \omega_S = \dfrac{2\pi}{T} = 400 \end{cases}$$

Find $X_r(t)$ for each ω_o given below (**explain the reasons, no score if guessing**)

(a) ω_o=100 (b) ω_o=150 (c) ω_o=300 (d) ω_o= 400 (e) ω_o=500