國立雲林科技大學 106 學年度 碩士班招生考試試題

系所：電機系

科目：工程數學(2)

1 (10 pts) Find the general solution of the differential equation: $y' + y = \sinh x$.

2 (10 pts) Find the general solution of the differential equation: $y^{(4)} + y = 0$. Note that $y^{(4)}$ denotes the fourth derivative of $y(x)$.

3 (10 pts) Solve the initial value problem: $x^2 y'' - 2xy' + 2y = 10\sin(\ln(x))$; $y(1) = 3$, $y'(1) = 0$. (Hint: this second order differential equation is an Euler's equation)

4 (10 pts) Find the inverse the Laplace transforms of (a) $F_1(s) = \dfrac{2s+7}{s^2+8s+25}$ and (b) $F_2(s) = \dfrac{e^{-2s}}{s(s+2)}$.

5 (10 pts) Use the Laplace transform to solve the system of differential equations

$$x'' - 2x' + 3y' + 2y = 2$$
$$2y' - x' + 3y = 0$$
$$x(0) = x'(0) = y(0) = 0$$

6 (15 pts) Find the Fourier half cosine and Fourier half sine expansions of $f(x)$ for $f(x) = \begin{cases} 1, & \text{for} \quad 0 < x < 1 \\ 2, & \text{for} \quad 1 \le x < 2 \end{cases}$

7 (10 pts) Find the Fourier cosine transform of $f(t) = te^{-at}$, $a > 0$.

8 (10 pts) Find values of a and b such that the system of linear equations has (a) no solution (b) exactly one solution and (c) infinity solutions.

$$x + 2y = 3$$
$$ax + by = -6$$

9 $\vec{\mathbf{X}}'(t) = \begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix} \vec{\mathbf{X}}(t) = \mathbf{A}\vec{\mathbf{X}}(t)$.

(8 pts) (a) Find (if possible) a nonsingular matrix **P** and **D** such that $\mathbf{D} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ is diagonal.

(7 pts) (b) Find the general solution of $\vec{\mathbf{X}}(t)$.