



- (15%) Solve the general solution of the following differential equations:  
[解下列微分方程式之通解]  
(1)  $y' = 2 \cos 2x$ ; (2)  $y' + 2xy = e^{-x^2}$ ; (3)  $(x-1)^2 y'' + 2(x-1)y' - 6y = 0$
- (10%) Solve the following initial value problem [解下列微分方程式之解]:  
 $y'' + 9y = 36e^{3x}$ ;  $y(0) = 0$ ,  $y(\pi/2) = 0$
- (15%) Find the Laplace transform of the following functions [求下列函數之拉氏轉換;  $F(s) = L[f(t)]$ ]:  
(1)  $f(t) = t^2 - e^{-9t} + 5$ ; (2)  $f(t) = \int_0^t \tau \sin \tau d\tau$ ; (3)  $f(t) = t^2 - e^{-t} - \int_0^t f(\tau)e^{-\tau} d\tau$
- (10%) Use the Laplace transform to solve the given system equations [利用拉氏轉換求解以下聯立方程式系統]:  

$$\begin{aligned} x' + \int_0^t y dt &= 1; & x(0) &= 0 \\ -4x + y &= 0 \end{aligned}$$
- (15%) 求解以下線性聯立方程式系統
 
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$$
 (1) 使用高斯喬登方法(Gauss-Jordan method); (2) 使用反矩陣方式求解;  
 (3) 使用克萊姆法則(Cramer's rule)。
- (15%) 使用 Gram-Schmidt 正交化程序，將  $\mathbf{w}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ ,  $\mathbf{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix}$ ,  $\mathbf{w}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$  產生正交基底 (orthogonal basis)  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$
- (10%) 令線性轉換  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  定義為以下數學式:  $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ -x_1 + x_2 \\ 2x_1 - x_2 \end{bmatrix}$   
 (1) 求矩陣  $A$  使得  $T(\mathbf{x}) = A\mathbf{x}$ ; (2) 決定此轉換的零數(Nullity)與秩(Rank).
- (10%) 矩陣  $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ ; (1) 求特徵值(Eigenvalues), (2) 求特徵向量(Eigenvectors), (3) 將矩陣  $A$  對角化 (Diagonalizing)。