



1. Given that the Laplace transform $\mathcal{L}\left\{\frac{2}{t}[1 - \cos(t)]\right\} = \ln\left(\frac{s^2 + 1}{s^2}\right)$,
please find the value of $\mathcal{L}\left\{\frac{1}{t}[1 - \cos(2t)]\right\}$ (10%)
2. Find the inverse Laplace transform for the following function. (10%)
$$\frac{se^{-s}}{(s+1)^2(s^2+2s+2)}$$
3. Find the general solution for the following differential equations. (30%)
- (a) $(D^4 + 5D^2 - 36)y(x) = 10e^{-2x} + 3\cos(3x)$. (10%)
- (b) $(x^3D^3 + 3x^2D^2 + xD - 1)y(x) = 0$. (10%)
- (c) $\frac{dy}{dx} = \frac{6xy - y^2}{3xy - 6x^2}$. (10%)
4. Find the Fourier half cosine and Fourier half sine expansions of $f(x)$ for
$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \end{cases}$$
 (15%)
5. Solve the following integral equation for the function $f(x)$
$$\int_0^\infty f(x) \sin(\omega x) dx = \begin{cases} 1, & 0 < \omega < 1 \\ 0, & \omega > 1 \end{cases}$$
 (10%)
6. Determine the polynomial $y = a_0 + a_1x + a_2x^2$ whose graph passes through the points (x, y) of $(1, 9)$, $(2, 18)$, and $(3, 31)$. (10%)
7. Consider the following linear equations $\mathbf{Ax}=\mathbf{b}$,
$$\begin{cases} x_1 & - 2x_3 + x_4 = 4 \\ 3x_1 + x_2 - 5x_3 & = 8 \\ x_1 + 2x_2 & - 5x_4 = -4 \end{cases}$$
- Write the solution in the form $\mathbf{x} = \mathbf{x}_h + \mathbf{x}_p$, where \mathbf{x}_h is the solution of $\mathbf{Ax}=\mathbf{0}$ and \mathbf{x}_p is a particular solution of $\mathbf{Ax}=\mathbf{b}$. (8%)
8. Let A be an 4×4 invertible matrix and $\text{adj}(A)$ be the adjoint of A . Find the value x of the determinant $|\text{adj}(A)| = |A|^x$. (7%)