



1. Apply Laplace transform to solve the equation,  
 $y''(t) + 2ty'(t) - 6y = t; y(0) = 0, y'(0) = 0$  (10%)

2. Find the inverse Laplace transform for the following function. (10%)

$$\frac{s}{(s+1)^2(s^2+2s+5)}$$

3. Apply Laplace transform to find the solution for the following equations. (10%)

$$x(t) + 3 \int_0^t [x(\tau) - y(\tau)] d\tau = 1$$

$$y(t) + 2 \int_0^t [2y(\tau) - x(\tau)] d\tau = 0$$

4. Find the Fourier transform for the following function. (10%)

$$\frac{3e^{it}}{t^2 - 2t + 5}$$

5. Find the inverse Fourier transform for the following function. (10%)

$$\frac{1}{(1+\omega^2)(4+\omega^2)}$$

6. Find the general solution for the following differential equations.

(i)  $y^2 + y - x \frac{dy}{dx} = 0$  (10%)

(ii)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 16 + (12x - 4)e^{2x}$  (15%)

7. Determine the relationship of  $a, b, c$  such that the following system of linear equations has
- (i) an infinite number of solutions, (5%)
  - (ii) exactly one solution, (5%)
  - (iii) no solution. (5%)

$$2x - y + z = a$$

$$x + y + 2z = b$$

$$3y + 3z = c$$

8. Let  $w = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -12 \end{bmatrix}$ ,  $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} -1 \\ -2 \\ -3 \\ 4 \end{bmatrix}$  and  $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ . Write the vector  $w$  as a

linear combination of vectors  $v_1, v_2$  and  $v_3$ . (10%)