



共 9 題，合計 100 分，請依序作答，否則不計分

1. Find the general solution of the differential equation (10 分)

$$\frac{dy}{dx} = -\frac{3xy + y^2}{x^2 + xy} \quad [\text{Hint: Exact Form- using integrating factor } \phi(x) = x]$$

2. Find the general solution of the differential equation: $y'' - 2y' - 8y = 6e^{-2x}$
(15 分)

3. The nonhomogeneous system of linear equations $AX = B$, in which

$$A = \begin{bmatrix} -1 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, \text{ Find (1) the reduced row echelon form of augmented matrix } [A|B], \text{ (2) the dependent unknowns and independent unknowns, and (3) the general solution of } AX = B. \text{ (15 分)}$$

4. Let $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$, find (1) the eigenvalues and eigenvectors of A, and (2) the matrix A^{10} . (10 分)

5. Apply Laplace transform to solve the equation,

$$y''(t) + 4y'(t) + 4y = 3H(t-2); y(0) = 0, y'(0) = 0,$$

where H(t) is Heaviside function. (10%)

6. Find the inverse Laplace transform for the following function. (10%)

$$\frac{3e^{-2s}}{(s+1)^2(s^2+2s+10)}$$

7. Apply Laplace transform to find the solution for the following equations. (10%)

$$x''(t) - 2x'(t) + 3y'(t) + 2y(t) = 3.$$

$$\dots\dots\dots 2y'(t) - x'(t) + 3y(t) = 0,$$

$$\dots\dots\dots x(0) = x'(0) = y(0) = 0.$$

8. Find the Fourier transform for the following function (10%)

$$f(t) = t[H(t+2) - H(t-2)],$$

where H(t) is Heaviside function.

9. Find the inverse Fourier transform for the following function (10%)

$$\frac{5e^{14\omega} \cos(2\omega)}{(9 + \omega^2)(4 + \omega^2)}$$