



1. Find the general solution of the differential equation (10 分)

$$[D^3 - 2D^2 + D]y = 2x; \quad [\text{Note: } D^n y = y^{(n)} = \frac{d^n}{dx^n} y]$$

2. Find the general solution,  $y(x) = c_1 y_1(x) + c_2 y_2(x)$ , of the differential equation  $x^2 y'' + xy' - y = 0$ ,  $x > 0$ . To explain if  $y_1, y_2$  are linear independent by Wronskain test. (15 分)

3. Let  $A = \begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix}$ , find: (1)  $P$ , and diagonal matrix  $D = P^{-1}AP$ ,  
 (2)  $(A^2 + 6A + 4I)^5 = ?$  (10 分)

4. To solve the initial value problem,  $\begin{cases} x_1' = 2x_1 - 10x_2 \\ x_2' = -x_1 - x_2 \end{cases}$ ,  $X(0) = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$ , by matrix methods. (15 分)

5. Find the Laplace transform for the following functions (10%)  
 $[\sin(t-1) + (t^2 - 2)]H(t-1)$

6. Find the inverse Laplace transform for the following functions.

(a)  $\ln[(s+2)/(s-1)]$ , (10%)

(b)  $\frac{se^{-2s}}{(s+2)^2(s^2+4s+8)}$ . (10%)

7. Find the sum of the series  $\sum_{n=1}^{\infty} (-1)^n / (4n^2 - 1)$ . (hint :expand  $\sin(x)$  in a Fourier cosine series on  $[0, \pi]$  and choose an appropriate value of  $x$ . (10%)

8. Find the inverse Fourier transform for function:  $\frac{2e^{(\omega-2)t}}{[2 + (\omega-2)i]}$ . (10%)