



1. 金融海嘯引發了歐洲一些國家的債信問題，而有所謂的 "PIGS"，請問 PIGS 代表什麼？(10%)
2. 主導國際經濟的強權國家，最早是所謂的 "G7"，後來又拉攏了一國成為 "G8"，金融海嘯後為了擴大它的影響力，增加成 "G20"，而實際互別苗頭者又是 "G2"，請分別列出這些國家。(20%)
3. 請以 "SWOT" 的精神來論述 "ECFA"。(20%)

4. 請根據下列短文，回答問項：

「澳紐銀行表示，近日南歐爆發主權債信危機，造成全球股市連續幾天重挫，突顯全球經濟復甦仍然脆弱，而國際資金的大幅流動也為金融市場帶來風險，增加全球經濟陷入二次衰退的可能性。但就台灣而言，台灣經濟自去年下半年起逐漸好轉，擁有大規模的經常帳順差與外匯存底，加上台灣政府負債占 GDP 約 40%，負債比仍在可控制水準，且相較其他許多國家來得低，因此，台灣的經濟體質和金融體系仍相當穩健。」

- (1) 請就「南歐爆發主權債信危機」的事件進行簡短的說明。(5%)
- (2) 「國際資金的大幅流動也為金融市場帶來風險」，該風險是指？(10%)
- (3) 「南歐爆發主權債信危機的事件」對歐元有何影響？(5%)
- (4) 歐洲是台灣第四大貿易夥伴，請分析此事件歐元的變動對台灣貿易商可能產生的影響為？(10%)

5. iPad 是 Apple 公司推出，廣受注目的新產品，請根據下列短文，以該電子產品經銷商的角度回答問項 (可用中文作答)：

**「Apple's iPad: No Fun for Small Biz**

Yet for me, as a small business owner who sells software and related consulting services, the iPad is destined to become a pain in the rear. Some of my clients are already talking about buying one. So are some of my employees.

Clients are going to ask me why I don't recommend it for their business. My employees are going to get frustrated when I tell them to leave their iPads at home. They're all going to call me names and accuse me of living in the Dark Ages. A few of them will probably make fun of my height, too, just like they did when I pooh-poohed netbooks.



They'll see everyone else playing with those iPads, and they'll also want one. But they won't understand its limitations.

Fun as it may be, the iPad will have its own unique support issues. And my 10-person company barely has the resources to support our existing devices—the PCs, Macs, Research In Motion (RIMM) BlackBerrys, the iPhones, the netbooks, etc. So when some employee of mine insists on using his new iPad and then runs into a printing issue or a connection problem, I'm going to have to step up and help him so he stays productive. Which means I'm going to have to incur yet more costs with my IT firm. The upshot: I'll be paying them to learn how to support the new iPad, because they'll be just as clueless. This won't be fun.

Because with that boring laptop, you'll have compatibility with most business software. In fact, right now the iPad only lets you install software from the Apple Store (but I'm sure that'll change in the future). With a boring laptop running boring Windows, you'll have USB ports. And replaceable batteries. And a full-size keyboard. You'll have a broader network of support from boring business partners that have years of experience with similar devices (which is why they're so boring). And you'll have more options for securing data going back and forth between your employees and your database, which contains boring business things like orders, invoices, and payroll.

O.K., so the iPad really won't reduce any costs in my business. Can we justify its business ROI through increased employee productivity? Ha! You try using one of those things for a few hours and then tell me how much actual work you accomplished. It's just too damn fun. 」

- (1) iPad 對電子產品經銷商的可能帶來的問題是? (10%)
- (2) 出售 iPad 對該經銷商績效的可能影響是? (10%)



1. A learning test was given at three cities. 1,200 students took the test at Taipei, 500 students at Taichung, and 800 students at Kaohsiung. The percentages of students from Taipei, Taichung, and Kaohsiung who passed the test were 50%, 60%, and 75%, respectively. One student is selected at random from among those who took the test.

- (a) What is the probability that the selected student passed the test? \_\_\_\_\_ (4 分)
- (b) If the selected student passed the test, what is the probability that the student took the test at Kaohsiung? \_\_\_\_\_ (3 分)
- (c) What is the probability that the selected student took the test at Taipei and passed? \_\_\_\_\_ (3 分)

2. If the joint density of  $X_1$ ,  $X_2$ , and  $X_3$  is given by

$$f(x_1, x_2, x_3) = \begin{cases} \frac{2}{3}(x_1 + x_2 + x_3) & 0 < x_1 < 1, 0 < x_2 < 1, 0 < x_3 < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a)  $P(X_1 \leq 0.5, X_2 \leq 0.5, X_3 \leq 0.5) =$  \_\_\_\_\_ (4 分)
- (b) Compute the variance of  $X_1$ . \_\_\_\_\_ (3 分)
- (c) Compute the mean of  $X_2$ . \_\_\_\_\_ (3 分)

3. A continuous random variable  $X$  has the following probability density function:

$$f(x) = a + bx \text{ for } 0 \leq x \leq 4. \text{ Furthermore, the expectation of } X \text{ is } \frac{4}{3}.$$

- (a) Find  $a =$  \_\_\_\_\_ and  $b =$  \_\_\_\_\_ (4 分)
- (b)  $P(X \leq 3) =$  \_\_\_\_\_ (2 分)
- (c)  $P(X \geq 1) =$  \_\_\_\_\_ (2 分)
- (d)  $P(1 \leq X \leq 3) =$  \_\_\_\_\_ (1 分)
- (e)  $P(X = 2) =$  \_\_\_\_\_ (1 分)



4. In order to estimate the mean income for a population of 8,000 households, the inspector selected at random a sample of 50 households.
- (a) Would you use the finite population correction factor in calculating the standard error of the sample mean? Explain. (3 分)
- (b) If the population standard deviation is \$10,000, compute the standard error both with and without using the finite population correction factor. (4 分)
- (c) What is the probability that the sample mean income of the households will be within  $\pm \$4,000$  of the population mean income? (3 分)
5. A random sample of 100 observations from a normal population whose standard deviation is 20 produced a mean of 40. Provide sufficient evidence at the 5% level of significance to infer that the population mean is not 50.
- (a) What is the null hypothesis? \_\_\_\_\_ (2 分)
- (b) What is the alternative hypothesis? \_\_\_\_\_ (2 分)
- (c) Test statistic  $Z =$  \_\_\_\_\_ (2 分)
- (d) What is the reject region? \_\_\_\_\_ (2 分)
- (e) Conclusion: \_\_\_\_\_ (2 分)
6. Let  $Y_1 < Y_2 < \dots < Y_{19}$  be the order statistics of a random sample of size  $n=19$  from the exponential distribution with mean  $\theta$ .
- (a) What is the p.d.f. of  $Y_1$ ? \_\_\_\_\_ (5 分)
- (b) Find the value of  $E[F(Y_1)]$ , where  $F$  is the distribution function of the exponential distribution. \_\_\_\_\_ (5 分)



7. We have some observations for  $(x,y)$ : (21,2.8), (24,3.4), (26,3.0), (27,3.5), (29,3.6),

(a) Estimate the slope of the equation  $y = \hat{\beta}_0 + \hat{\beta}_1 x$  is \_\_\_\_\_? (5 分)

(b) The  $R^2$  of this equation is \_\_\_\_\_? (5 分)

8. For the regression in question number 7(a),

(a)  $\sum_{i=1}^5 \hat{u}_i$  is \_\_\_\_\_? (where  $\hat{u}$  is the residual) (5 分)

(b)  $\sum_{i=1}^5 x_i \hat{u}_i$  is \_\_\_\_\_? (5 分)

9. If we estimate a linear model

$$Y_i = \beta_0 + \beta_1 X_i + U_i, i=1, \dots, n$$

However, the real model is  $Y_i = \beta_1 X_i + V_i$ , where the mean of  $V_i$  is 0 and the

variance is  $\sigma_0^2$ , also,  $\text{cov}(V_i, V_j) = 0$ . What is the variance of OLS estimator of

$\beta_1$ ? \_\_\_\_\_ (10 分)

10. For a linear model

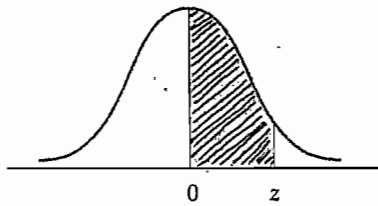
$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_5 X_5 + U$ . If  $n=50$  and  $R^2 = 0.7$ , the value of  $\bar{R}^2$  is \_\_\_\_\_? (10 分)



B-8 Appendix B

Table 3

Normal Probabilities



| z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |

SOURCE: Abridged from Table 1 of A. Hald, *Statistical Tables and Formulas* (New York: Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher, John Wiley & Sons, Inc.



本試題共兩部分，第一部分為單選題 6 題，每題 6 分共 36 分，第二部份為填充題 16 小題，每小題 4 分共 64 分。兩部份合計 100 分。請依題目順序將答案寫在答案卷上，違者不予計分。

第一部分 選擇題 (每題 6 分, 共 36 分)

以下 6 題皆為單選題，請在選項中選出一個最正確的答案。

1. Consider the following statements regarding price elasticity:

I. Because the demand of rice is inelastic, farmer's income could be lower in the year of abundant harvest.

II. Because the demand of drug is inelastic, a policy which reduces drug demand is a better way to reduce drug-related crime than a policy which reduces drug supply.

III. Because the long-run-elasticity of oil demand is greater than the short-run one, it is harder for the Organization of the Petroleum Exporting Countries (OPEC) to maintain a high oil price by reducing oil production in the long-run.

- (a) Only statement I is correct.
- (b) Only statement II is correct.
- (c) Only statement III is correct.
- (d) Only statement I and II are correct.
- (e) Only statement II and III are correct.
- (f) Only statement I and III are correct.
- (g) All of the three statements are correct.

2. The demand for MBA and DBA increases rapidly in U.S. corporate during 1990s, leading to an increase in the salary of MBA and DBA. Please consider the impact of raising salary of MBA and DBA on the labor market of professors in the management school.

I. The increasing salary of MBA and DBA in U.S. corporate will lead to an increase in the supply of professors in the management school.

II. The increasing salary of MBA and DBA in U.S. corporate will lead to an increase in the demand of professors in the management school.

III. The increasing salary of MBA and DBA in U.S. corporate will cause changes in labor market and lead to higher salary for professors in the management school.

IV. The increasing salary of MBA and DBA in U.S. corporate will cause changes in labor market and lead to higher employment for professors in the management school.

- (a) Only I and IV are correct.
- (b) Only II and III are correct.
- (c) Only II and IV are correct.



- (d) Only I and III are correct.  
 (e) Only II, III, IV are correct.  
 (f) Only I, II, III are correct.  
 (g) All of the above statements are correct.

3. 請考慮以下關於物價指數的敘述：

- I. 若某一年度的實質 GDP 大於名目 GDP, 則該年以 GDP 平減指數計算的物價膨脹率將小於零。  
 II. 同一年度中, 根據 GDP 平減指數所計算出的物價膨脹率與根據消費者物價指數 (CPI) 所計算出的物價膨脹率兩者未必相同, 但是當前者大於零時後者也會大於零, 反之亦然。  
 III. 由於消費者物價指數 (CPI) 沒有考慮產品品質的改變, 所以必然高估消費者的生活成本。  
 IV. 當政府改變計算消費者物價指數 (CPI) 的基期時, 雖然各年度的 CPI 將會隨之改變, 但是用新的基期所計算出的物價膨脹率將與以舊的基期計算出來的數字相同。

- (a) 僅 III 正確。  
 (b) 僅 IV 正確。  
 (c) 僅 I, IV 正確。  
 (d) 僅 II, III 正確。  
 (e) 僅 III, IV 正確。  
 (f) 僅 II, IV 正確。  
 (g) 所有敘述皆不正確。

4. 考慮以下的賽局, 括弧裏面的數字分別代表廠商 1 與廠商 2 的利潤, 單位為千萬:

|      |      | 廠商 2  |       |       |
|------|------|-------|-------|-------|
|      |      | 策略 A  | 策略 B  | 策略 C  |
| 廠商 1 | 策略 A | (9,4) | (2,7) | (8,6) |
|      | 策略 B | (3,1) | (8,4) | (7,3) |
|      | 策略 C | (7,7) | (6,3) | (5,4) |

- I. 此賽局中兩廠商都沒有優勢策略。  
 II. 請問此賽局只有一個單純策略 (pure strategy) NASH 均衡。  
 III. 在兩廠商都不能使用策略 B 的單純策略 NASH 均衡中, 廠商 2 的報酬將比可以使用策略 B 的單純策略 NASH 均衡的報酬來的高。  
 IV. 在只考慮單純策略之下, 假設某政府高官有權決定兩廠商能不能使用策略 B 來競爭, 此時廠商 1 會想遊說該政府高官禁止使用策略 B。  
 V. 在只考慮單純策略之下, 假設某政府高官有權決定兩廠商能不能使用策略 B 來競爭, 此時想要遊說該高官禁止使用策略 B 的廠商最多願意出 3 千萬的價錢來收買該高官?  
 請問以上的敘述:





- (a) 僅 I 正確。
- (b) 僅 II 正確。
- (c) 僅 I, II 正確。
- (d) 僅 I, II, III 正確。
- (e) 僅 I, II, III, IV 正確。
- (f) 僅 I, II, III, V 正確。
- (g) 所有敘述皆正確。

5. 考慮在完全競爭市場下某製鋁工廠的短期生產成本。由於廠房機器設備在多年前就已建造完成，為簡化起見，假設該工廠沒有固定成本。若該工廠採 16 小時兩班制運作，每天最大產量為 1000 噸，此時每噸原料成本為 500 元，另有人力及維護成本為 500 元。若產量超過 1000 噸，工廠必須增開大夜班，大夜班的最大產量為 400 公噸，每噸原料成本仍為 500 元，但因支付加班費的緣故，人力及維護成本每噸增加為 1000 元。請根據以上資訊考慮下面的敘述：

- I. 產量在 1000 噸以內時，生產每公噸鋁的邊際成本等於平均成本。
- II. 產量為 1200 噸時，生產鋁的邊際成本為 1,500。
- III. 產量為 1200 噸時，生產鋁的平均成本將高於邊際成本。
- VI. 若市場價格為每噸 1,200 元，該工廠最適的產量為 1,000 噸。
- V. 若市場價格為每噸 1,500 元，該工廠最適的產量為 1,500 噸。

- (a) 僅 II, VI 正確。
- (b) 僅 I, III, V 正確。
- (c) 僅 I, II, IV 正確。
- (d) 僅 I, II, V 正確。
- (e) 僅 II, III, IV, V 正確。
- (f) 僅 I, II, IV, V 正確。
- (g) 所有敘述皆正確。

6. 假設某國過去平均的實質 GDP 成長率為 4%，貨幣供給成長率為 6%，名目利率固定在 3% 的水準。該國新任央行總裁認為未來的 GDP 成長率會下降到 1%。該國的貨幣需求函數為  $-(M^d/P) = A*(Y/R)$ 。其中 P 為物價指數，Y 為實質 GDP，R 為固定不變的名目利率，A 是一個固定的常數。

- I. 在 GDP 成長率為 4% 之下，根據貨幣需求函數可知該國的物價膨脹率為 2%。
- II. 當 GDP 成長率下降到 1% 時，若貨幣供給成長率維持在 6%，此時物價膨脹率將提高為 5%。
- III. 假設央行總裁希望在 GDP 成長率下降後維持物價膨脹率不變，則應該將貨幣供給成長率調降為 3%。
- IV. 假設經濟成長率下降時連帶使得 A 的值變為原來的 1/2，則央行總裁將貨幣供給成長率調維持在 6% 就可以使得 GDP 成長率下降後維持物價膨脹率不變。



- (a) 僅 I 正確。
- (b) 僅 IV 正確。
- (c) 僅 I, IV 正確。
- (d) 僅 II, III 正確。
- (e) 僅 II, III, IV, 正確。
- (f) 僅 I, II, III 正確。
- (g) 所有敘述皆正確。

第二部分 填充題 (共 16 小題, 每小題 4 分, 合計 64 分)

◎ 開心市周邊有 2 家工廠, 每家所排放的汙染與每單位的汙染防治成本如下表所示:

| 工廠 | 汙染量 | 每單位汙染防治成本 |
|----|-----|-----------|
| A  | 90  | 25        |
| B  | 50  | 15        |

由於居民抱怨汙染嚴重, 政府希望能夠將總汙染排放量降到 80 單位。請考慮以下敘述:

- (1) 假設政府發放每家工廠 40 單位的汙染排放許可, 在不能私底下交易排放許可的情況下, 請求出此時 2 家工廠的總汙染防治成本。
- (2) 假設政府允許廠商間交易排放許可, 此時 A 工廠會跟 B 工廠購買汙染排放許可。(請回答「是」或「非」)
- (3) 假設政府允許廠商間交易排放許可, 請問在交易之後 B 工廠的汙染排放量為多少?
- (4) 假設政府允許廠商間交易排放許可, 在交易之後 2 家工廠的總汙染防治成本將變成多少?

◎ Mary is considering how to deal with her New Year's money in the red envelope. She plans to save a fraction of her money in the bank and earns a risk-free rate of 5%. The remaining of her money will be used to purchase common stocks of company F. The expected rate of return for the stock is 10% and the standard deviation of the return is 20%. Let  $x$  be the fraction of money that Mary deposit in the bank. For simplicity, we assume  $0 \leq x \leq 1$ . Suppose that Mary's preference on the investment portfolio can be represented by a mean-variance utility function  $U(\mu, \sigma) = 40\mu - 0.5k\sigma^2$ , where  $\mu$  is the portfolio's rate of return,  $\sigma$  is the portfolio's standard deviation, and  $k$  is a constant. Mary will choose  $x$  to maximize her utility.

- (5) Suppose  $k=1$ , find the optimal  $x$  for Mary.
- (6) Suppose  $k=2$ , then Mary will save more than 50% of her money in the bank. (Please answer "True" or "False")
- (7) Assume  $k=1$ . Suppose the expected rate of return for the stock increases to 20% and the standard deviation of the return becomes 40%, find the optimal  $x$  for Mary.
- (8) Assume  $k=1$ . Suppose the risk-free rate increases to 8%. All other things being equal,



Mary will save more than 50% of her money in the bank. (Please answer “True” or “False”)

◎ 某獨占廠商面對一條直線型負斜率的市場需求線，假設該廠商生產沒有固定成本，每單位產品的邊際成本固定為 4 元。在該廠商追求利潤最大之下，該產品的售價為 7 元，此時的銷售量為 6 單位，而消費者剩餘為 9 元。另外，在上述的情況下，使社會福利最大的產量為 12，在該產量之下消費者剩餘為 36 元，但是廠商的利潤為 0。請考慮以下敘述：

- (9) 在廠商訂定的單一價格 (7 元) 之下，請問總利潤為何？
- (10) 在廠商訂定的單一價格之下，請問社會的無謂的損失為何？
- (11) 假設該廠商可以在沒有額外的成本之下進行完全差別訂價，此時廠商的總利潤為何？
- (12) 假設進行完全差別訂價須要花費成本  $C$ ，則當  $C$  小於或等於 18 時廠商才會願意進行完全差別訂價。(請回答「是」或「非」)

◎ 某廠商生產 X 與 Y 兩種產品，其面對四類型的消費者，每類消費者的人數相同。假設生產沒有任何成本，廠商追求利潤最大，各類型消費者對於 X 與 Y 產品的保留價格如下表所示：

|      | A 型消費者 | B 型消費者 | O 型消費者 | AB 型消費者 |
|------|--------|--------|--------|---------|
| 產品 X | 30     | 55     | 60     | 110     |
| 產品 Y | 110    | 100    | 90     | 30      |

- (13) 在兩種產品分別訂價及銷售之下，請問廠商對於 X 產品的最適訂價為何？
- (14) 在兩種產品分別訂價及銷售之下，請問在最適訂價之下 Y 產品的利潤為何？
- (15) 請問若廠商將兩種產品搭售(bundling)，則搭售價格應訂為多少？
- (16) 請問本題中將兩產品搭售的總利潤將比分別訂價及銷售的總利潤增加多少？



1. Prove that  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 5, & x = 0 \end{cases}$  is not continuous at  $x=0$ . Can one redefine  $f(0)$  so that  $f(x)$  is continuous at  $x=0$ ? (10 分)
2. Prove that  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ . (5 分)
3. Given  $\frac{d}{dx}(\sin x) = \cos x$  and  $\frac{d}{dx}(\cos x) = -\sin x$ , derive the formula:  
 $\frac{d}{dx}(\tan x) = \sec^2 x$ . (5 分)
4. Prove that  $\left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx$  if  $a < b$ . (5 分)
5. Find the relative maxima and minima of  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ . (5 分)
6. Show that the series whose  $n$ th term is  
 $u_n = \sqrt{n+1} - \sqrt{n}$  diverges although  $\lim_{n \rightarrow \infty} u_n = 0$ . (10 分)
7. Investigate the convergence of (a)  $\sum_{n=1}^{\infty} n^4 e^{-n^2}$  and (b)  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^n}{n^2}$ . (10 分)
8.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{4n^2 - k^2}} = ?$  (10 分)
9. Compute the volume of a solid generated by revolving the region bounded by the graphs of  $y = x$ ,  $y = x^2$ ,  $x = 2$ , and  $x = 5$  about the  $x$ -axis. (10 分)
10. A company sells two products whose demand functions are given by  $x_1 = 10 - p_1$  and  $x_2 = 3 - 0.2p_2$ . So, the revenue  $R = p_1 x_1 + p_2 x_2$ . Compute the revenue if  $x_1$  varies between 0 and 3 and  $x_2$  varies between 0 and 2. (10 分)
11. Solve the differential equation  $\frac{dy}{dx} = \ln x + x e^x$  with  $y(1) = 0$ . (10 分)
12. A graph satisfies the equation  $x^3 e^y + xy - 2e^x + y^2 - 2 = 0$ . Find the slope of the tangent line at the point  $(0, 2)$  of the graph. (10 分)