



1. (15%) Solve the general solution of the following differential equations:  
 [解下列微分方程式之通解]
  - (1)  $y' = 5 \sin 3x$  (5%); (2)  $y' + 5x^4 y = 3x^2 e^{-x^5}$  (5%); (3)  $x^2 y' = y^2 + xy$  (5%).
2. (10%) Solve the following initial value problems:  $y'' + 9y = 16 \sin x$ ;  $y(\pi) = 0$ ,  $y'(0) = 0$  (10%)
3. (15%) Use the Laplace transform or inverse transform to solve the given problems:
  - (1)  $f(t) = t^2 + e^{-6t} + \sin 6t$ ; 求  $F(s) = L[f(t)]$  (5%);
  - (2)  $F(s) = \frac{8}{s(s-2)^2}$ ; 求  $f(t) = L^{-1}[F(s)]$  (5%);
  - (3)  $y'(t) + \int_0^t y(\tau) d\tau = 2 - 2 \sin t$ ,  $y(0) = 0$ ; 求  $y(t)$  (5%).
4. (10%) Use the Laplace transform to solve the given initial value problem [利用拉氏轉換求解微分方程式]:  $\frac{dy(t)}{dt} - 3y(t) = te^{3t} \sin t$ ,  $y(0) = 0$ . (10%)
5. (20%) Let  $A = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ .
  - (a) Find the eigenvalues and associated eigenvectors (10%)
  - (b) Factor  $A$  into a product  $XD X^{-1}$ , where  $D$  is a diagonal matrix, and then use the factorization to compute  $A^5$ . (10%)
6. (10%) Determine whether the set  $S$  is linearly independent or linearly dependent.
  - (a)  $S = \{(-4, 3, 4), (1, -2, 3), (6, 0, 0)\}$  (5%)
  - (b)  $S = \{(6, -7, 8, 6), (4, 6, -4, 1), (2, 19, -16, -4)\}$  (5%)
7. (10%) Consider the matrix  $A = \begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ -2 & -8 & -4 & -2 \end{bmatrix}$ , find the null space, rank, and nullity of  $A$ . (10%)
8. (10%) For a matrix  $A = \begin{bmatrix} 1 & 0 & 1 \\ 7 & 7 & 8 \\ 1 & 2 & 1 \\ 7 & 7 & 6 \end{bmatrix}$ , apply the Gram-Schmidt process to find an orthonormal basis for the column space of  $A$ . (10%)



1. Find the equivalent capacitance  $C_{eq}$  in the circuit shown in Fig. 1. (10%)
2. Find voltage  $V_x$  and current  $I_x$  in the circuit of Fig. 2. (20%)

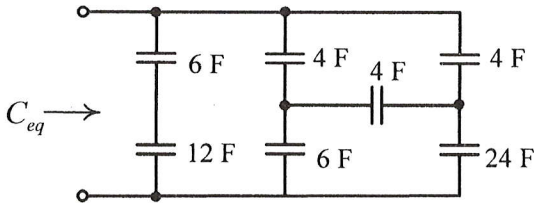


Fig. 1

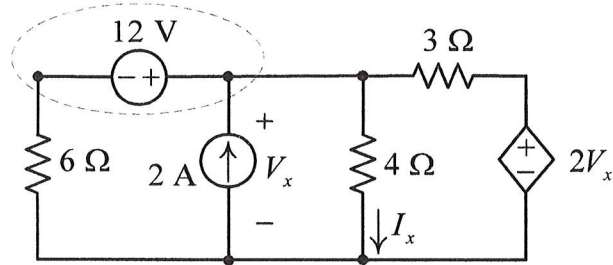


Fig. 2

3. In the circuit of Fig. 3, the sinusoidal voltage source is given by  $v_s(t) = 10\sqrt{2} \sin(10t)$  V. The switch closes at  $t=0$ . Find the current  $i(t)$  for  $t>0$ . (20%)
4. The current waveform in Fig. 4 is flowing through a 2-Ω resistor. Determine the rms value of the waveform and the average power delivered to the resistor. (16%)

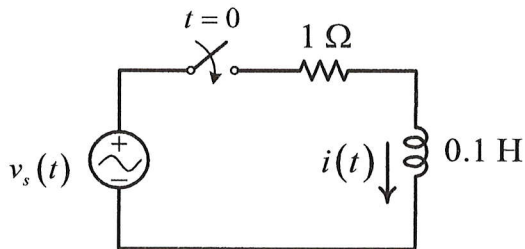


Fig. 3

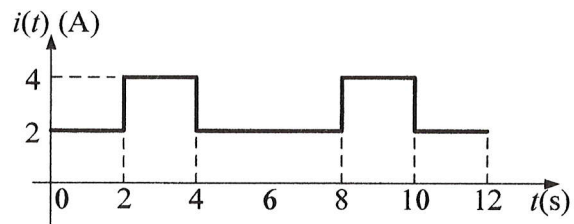


Fig. 4

5. In the balanced three-phase system shown in Fig. 5, the line voltage is 13.8 kV rms. Find the values of the capacitors  $C$  such that the total load has a power factor of 0.95 lagging. (16%)
6. Realize the transfer function  $\frac{V_o(s)}{V_i(s)} = \frac{-2s}{s+10}$  using the circuit in Fig. 6. Determine  $C_1$  and  $C_2$ . (18%)

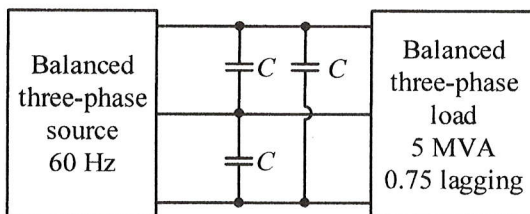


Fig. 5

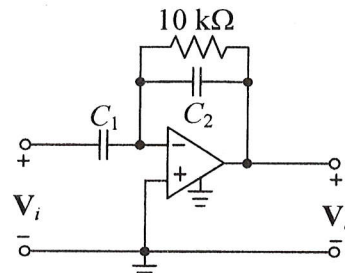


Fig. 6